Automatic Gain Control and Kicker Feedback Analysis for the Tune Tracker Phase Locked Loop (PLL)

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ABSTRACT: The automatic gain control (AGC) and the kicker feedback (KF) are two essential feedback loops in the tune tracker phase locked loop system. Unfortunately, these two systems if designed incorrectly can in principle fight each other and thus produce a loop that is far from ideal. We will propose a combined AGC and KF loop that ensures that both parts will be optimal. Both analytic solutions and computer simulations will also be discussed.

INTRODUCTION

Contained in the tune tracker phase locked loop (PLL) system[1] are two important feedback loops which keep the SQNR (signal to quantized noise ratio) high for the digital analogue converters. These two loops are the automatic gain control (AGC) and the kicker feedback (KF) loops. If these two loops are designed incorrectly, they have the potential of cancelling out each other and thus produce results that will be far from ideal. We will propose a way of decoupling these two loops by defining the working range for each of them.

We will analyze the AGC and the KF independently first and then combine them together to form an ideal multi-loop system. Note that AGCs are essentially non-linear systems but for particular types of variable gain amplifiers (VGA) and loop filters, there is an analytic solution[2]. This solution can also be easily extended for solving KF and so we will use this for the KF also.

Finally, we will combine the AGC and KF together and define their working ranges. With the working range defined, we can simulate their behaviour as the beam response changes. The simulation confirms that by partitioning the working range of the AGC and KF, they will work optimally.

The block diagram of one possible AGC implementation is shown in Figure 1. The input voltage V_{in} of the AGC for the tune tracker is usually a sinusoid i.e.

$$V_{\mathrm{ain}}(t) = V_{ai} \sin \omega t \tag{1}$$

where ω is the frequency of the tune tracker kicker.

The output voltage V_{aout} of the AGC is

$$V_{\mathrm{aout}}(t) = V_{ao} \sin(\omega t + \phi) \tag{2}$$

where V_{ao} is the peak output voltage and we have introduced ϕ, a phase shift between the input and the output because the AGC can introduce such a shift. In general, when the input voltage V_{ai} is in some finite range between V_{a1} and V_{a2}, the AGC keeps $V_{ao} \approx kV_{ar}$ where V_{ar} is the reference voltage set by the user and k is some constant.

Looking at Figure 1, we start the analysis at the variable gain amplifier (VGA). The VGA changes its gain g_a according to the control voltage V_{ac}. We choose the following equation to govern this change because we want to have an analytic solution of the AGC loop afterwards

$$g_a(V_{ac}) = G_a e^{-\alpha_a V_{ac}} \tag{3}$$

where $G_a > 0$ and $\alpha_a > 0$ are constant parameters of the VGA.

The rms detector converts its sinusoidal input into a constant voltage that is proportional to the rms value of the input, i.e.

$$V_{\mathrm{arms}} = V_{ao} \sqrt{\frac{1}{T_a} \int_0^{T_a} dt \ \sin^2(\omega t + \phi)} \tag{4}$$

where the integration time is chosen so that $T_a \gg 2\pi/\omega$. For large enough T_a, the argument under the square root of (4) is approximately equal to 1/2 and thus

$$V_{\mathrm{arms}} \approx \tfrac{1}{\sqrt{2}} V_{ao} \tag{5}$$

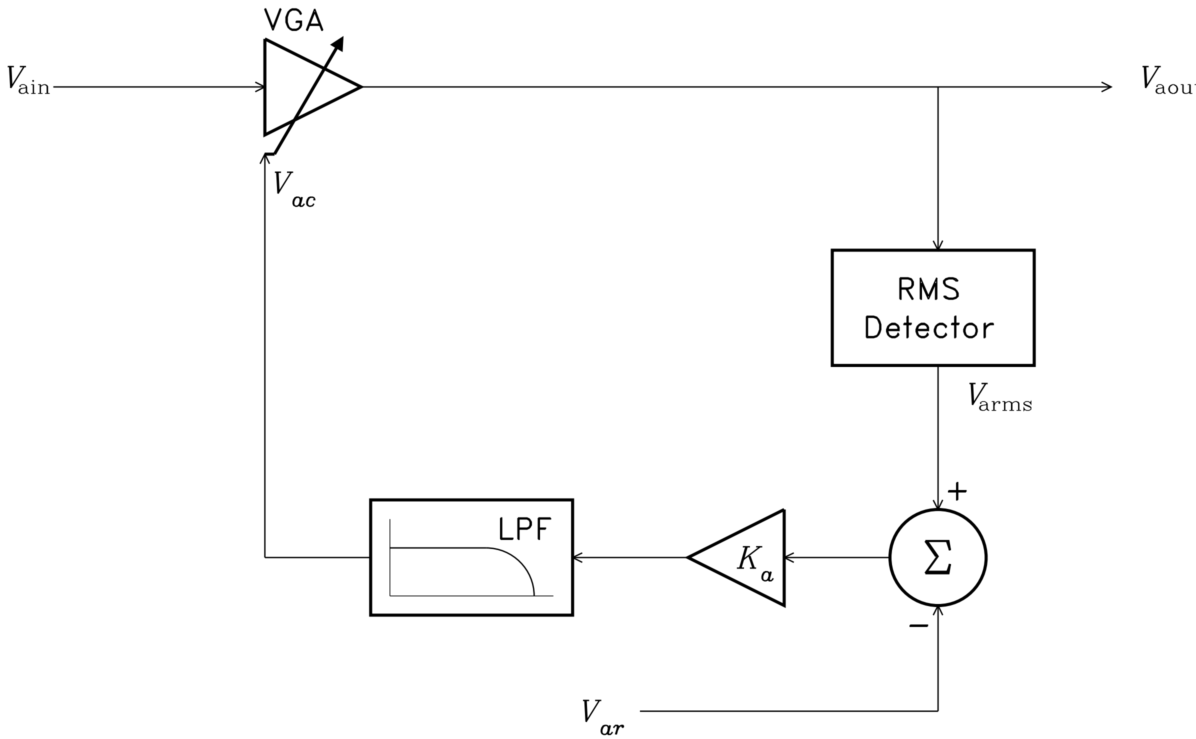

Figure 1 An implementation of the AGC. This type of AGC is similar to the one described in the paper "The Tevatron Tune Tracker PLL — Theory, Implementation and Measurements"[1].

This equation, (5), gives us an opportunity to simplify our approach to the problem: instead of considering sinusoids as the input and output, we can instead just calculate the behaviour of the AGC with the peak voltage of the sinusoids, i.e. V_{ai} and V_{ao}. In fact, V_{ai} and V_{ao} can be time dependent and they are the envelope of the carrier sinusoid. The simplified system that we will analyze is shown in Figure 2.

The analysis now continues along the lines of Ohlson[2]: the lowpass filter (LPF) of Figure 1 will be approximated with an integrator which starts integrating at $t = 0$ with $v_{a0} = v_a(0)$. Next, we proceed by finding V_{ao} and $v_a(t)$ in terms of V_{ai}. From (3), we can take the first derivative w.r.t. t to get

$$\dot{g}_a = -\alpha_a g_a \dot{v}_a \qquad \Rightarrow \dot{v}_a = -\frac{\dot{g}_a}{g_a \alpha_a} \tag{6}$$

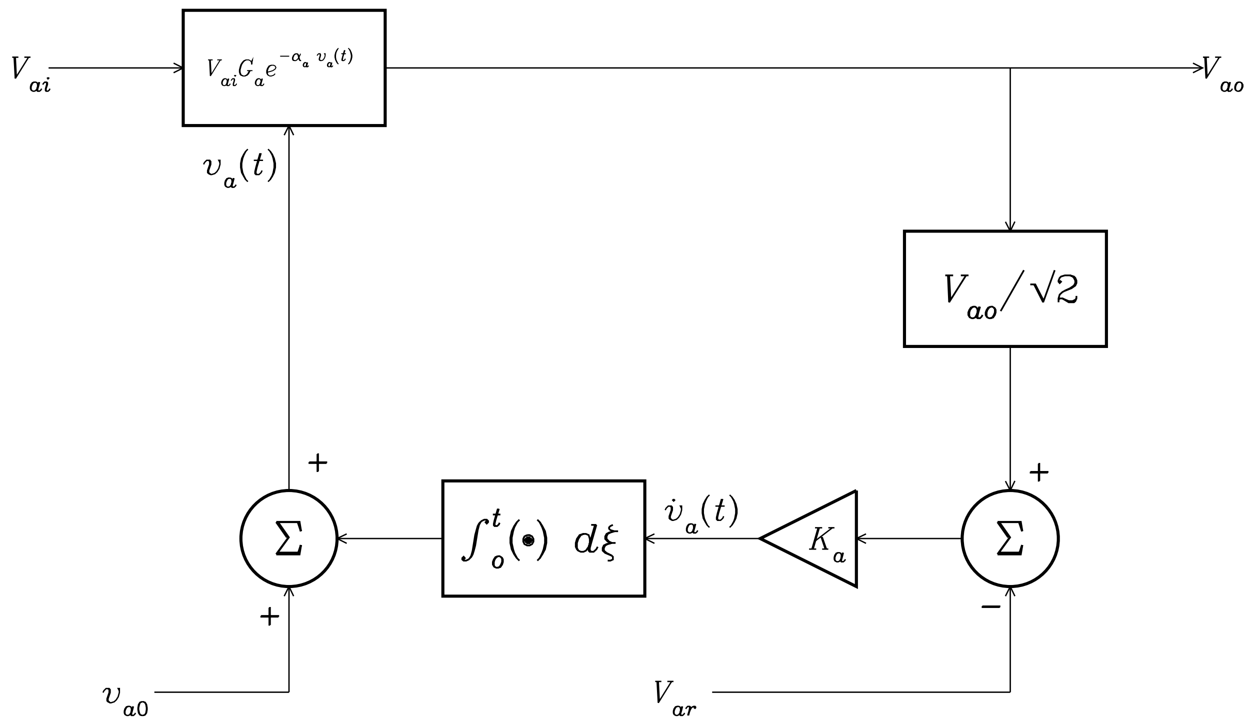

Figure 2 The equivalent AGC circuit that we can solve analytically. Notice that we only need to consider the peak values of the sinusoid.

where we have made the identification $V_{ac} \to v_a$ and "·" is the time derivative.

Next, as we have indicated in Figure 2,

$$\dot{v}_a = K_a \left(\frac{1}{\sqrt{2}} V_{ao} - V_{ar} \right) \tag{7}$$

If we go through the integrator, we will recover $v(t)$. However, this will not give us $v(t)$ in terms of V_{ai} and so we will do it via an indirect route. We substitute (6) into (7) to find that

$$\dot{g}_a + K_a g_a \alpha_a \left(\frac{1}{\sqrt{2}} V_{ao} - V_{ar} \right) = 0 \tag{8}$$

But $V_{ao} = g_a V_{ai}$ and thus (8) becomes

$$\dot{g}_a + \frac{1}{\sqrt{2}} K_a \alpha_a V_{ai} g_a^2 - K_a \alpha_a V_{ar} g_a = 0 \tag{9}$$

This differential equation is known as Bernoulli's equation and has the following solution

$$\left.\begin{aligned} g_a[v_a(t)] &= \left[\frac{e^{-t/\tau_a}}{g_a(v_{a0})} + \frac{1}{V_{ar}\sqrt{2}}\int_0^t d\xi \; \frac{1}{\tau_a}e^{-(t-\xi)/\tau_a}V_{ai}(\xi)\right]^{-1} \\ &= \left[\frac{e^{\alpha v_{a0}-t/\tau_a}}{G_a} + \frac{1}{V_{ar}\sqrt{2}}h_a * V_{ai}\right]^{-1} \end{aligned}\right\} \tag{10}$$

where $\tau_a = 1/K_a\alpha_a V_{ar}$, "$*$" is the convolution operator and we have defined h_a to be

$$h_a(t) = \begin{cases} \frac{1}{\tau_a}e^{-t/\tau_a} & t \geq 0 \\ 0 & t < 0 \end{cases} \tag{11}$$

Finally, $v_a(t)$ is found by solving for it by taking logarithms of (3) and substituting in (10) for g_a

$$v_a(t) = \frac{1}{\alpha_a}\log\left[e^{\alpha_a v_{a0}-t/\tau_a} + \frac{G_a}{V_{ar}\sqrt{2}}h_a * V_{ai}(t)\right] \tag{12}$$

V_{ao} is found by using $V_{ao} = g_a V_{ai}$ and thus

$$V_{ao}(t) = V_{ai}\left[\frac{e^{\alpha_a v_{a0}-t/\tau_a}}{G_a} + \frac{1}{V_{ar}\sqrt{2}}h_a * V_{ai}\right]^{-1} \tag{13}$$

Example

The time response with two steps in V_{ai} for this AGC when $G_a = 4$, $K_a = 300$, $\alpha_a = 1$, $v_{a0} = 0$ and $V_{ar} = 0.5$ is shown in Figure 3. From this, we see that at steady state V_{ko} is at $V_{ar}\sqrt{2} = 0.707$. The error which is $(V_{ao}/\sqrt{2} - V_{ar})$ is also zero at steady state. This is exactly how we expect the AGC to work.

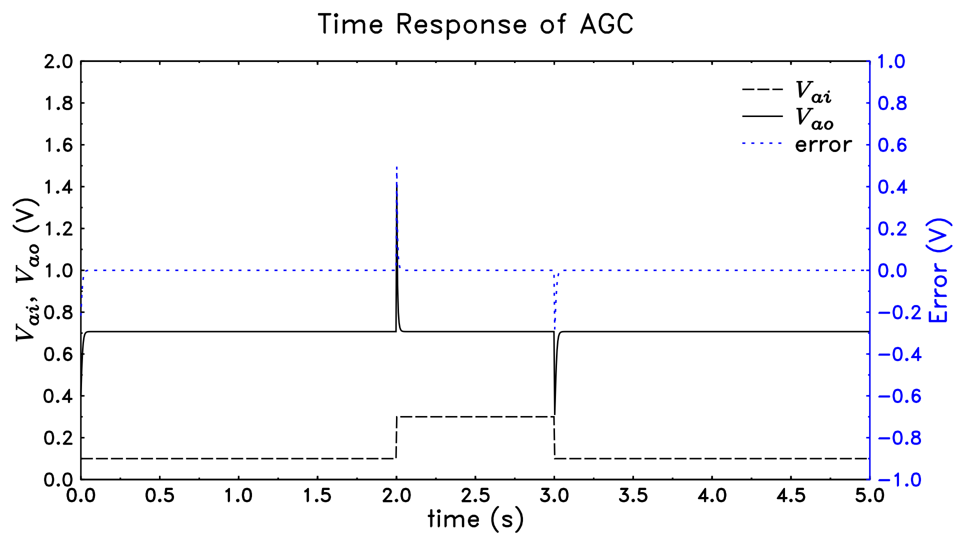

Figure 3 This shows the time response of the AGC when there are two steps in V_{ai}. The steady state value of V_{ko} is expected to be at $V_{ar}\sqrt{2} = 0.5 \times \sqrt{2} = 0.707$ and the error is also zero at steady state.

KF

The block diagram of an implementation of KF is shown in Figure 4. In the analysis of the KF, we can borrow many of the equations from the AGC section by simply remapping $a \rightarrow k$.

We will use the same VGA, and so its gain g_k varies according to the control voltage V_{kc}

$$g_k(V_{kc}) = G_k e^{\alpha_k V_{kc}} \tag{14}$$

The source of the kicker sine wave V_{in} is set at a fixed amplitude, i.e. V_{ki} = constant.

$$V_{\text{kin}} = V_{ki} \sin \omega t \tag{15}$$

The sine wave after the VGA is used to kick the beam. The measured amplitude of the sine wave after going through the beam is dependent on the frequency ω of the kick. If the

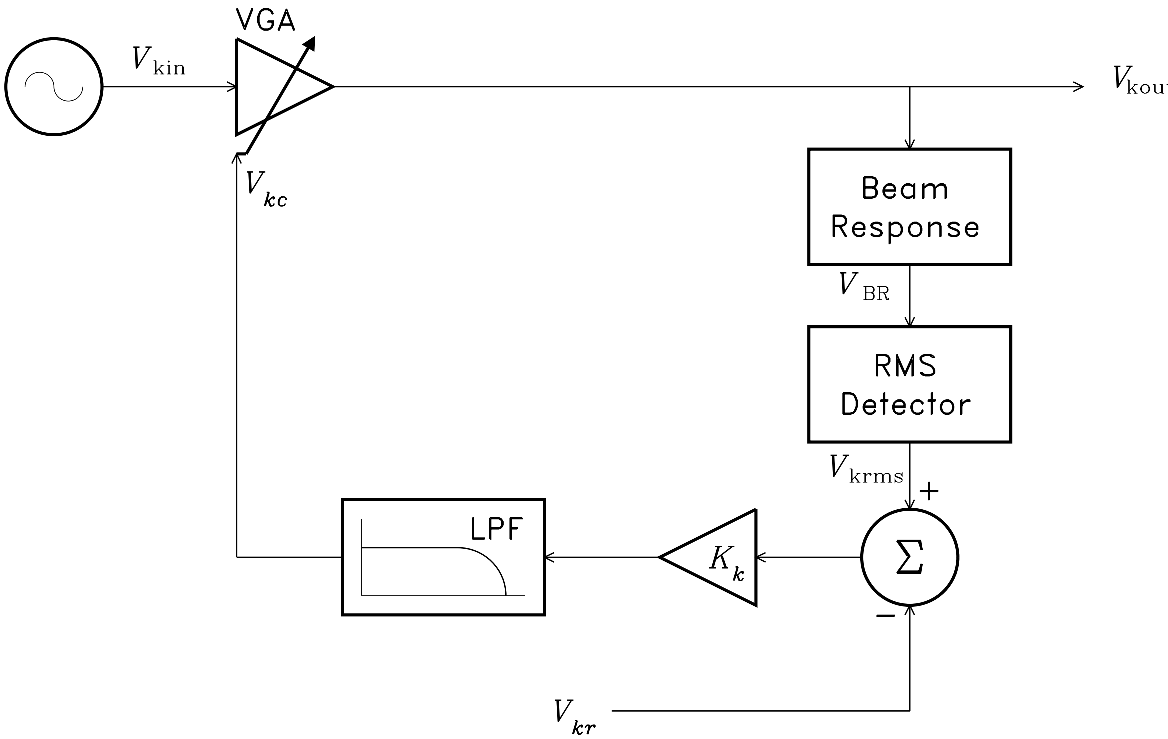

Figure 4 An implementation of the KF.

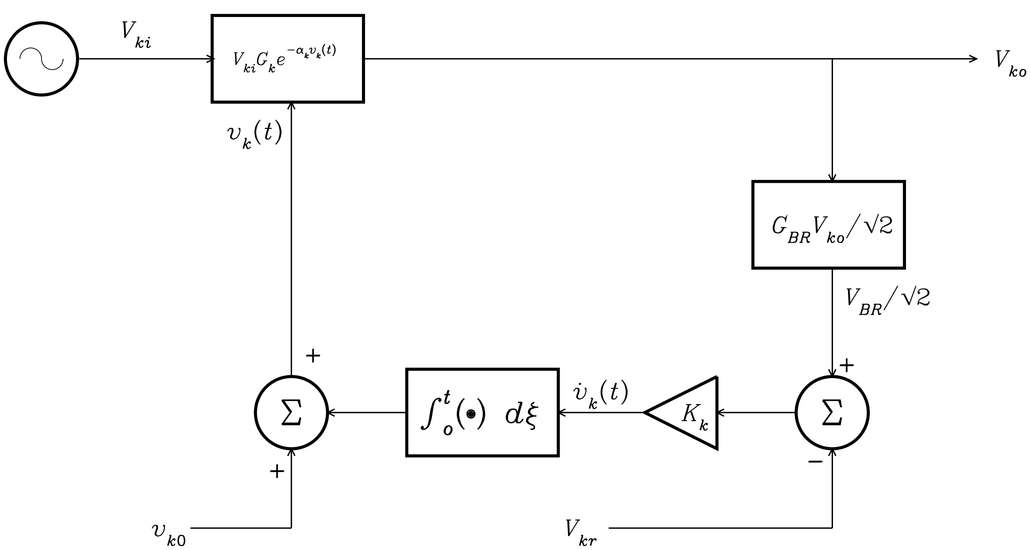

Figure 5 The equivalent KF circuit that we can solve analytically. Note the similarity to the AGC circuit shown in Figure 2.

beam has a transfer function $\tilde{G}_{BR}(\omega)$, then the measured amplitude is $\sim |V_{ki}\tilde{G}_{BR}|$. Thus if we define G_{BR} to be the "gain" from the beam, then we can define a new variable V_{BR} which is the voltage seen at the pickup to be

$$V_{BR} = V_{ki} g_k(V_{kc}) G_{BR}(t) \tag{16}$$

Note that G_{BR} is in principle complex, but the rms detector is blind to phase and thus for our purposes, we will let $G_{BR} \in \mathbb{R}$. Furthermore, G_{BR} is also time dependent because as the tune tracker PLL moves closer (or further) from resonance, G_{BR} increases (or decreases).

We continue our analysis of Figure 5,

$$\dot{v}_k = K_k \left(\tfrac{1}{\sqrt{2}} V_{BR} - V_{kr} \right) \tag{17}$$

and in the spirit of the previous section, we substitute (6) with $a \to k$ into (17), we have

$$\dot{g}_k + K_k g_k \alpha_k \left(\tfrac{1}{\sqrt{2}} V_{BR} - V_{kr} \right) = 0 \tag{18}$$

We substitute (16) into (18) to get Bernoulli's equation again

$$\dot{g}_k + \tfrac{1}{\sqrt{2}} K_k V_{ki} \alpha_k G_{BR} g_k^2 - K_k \alpha_k V_{kr} g_k = 0 \tag{19}$$

And thus, the solutions are

$$v_k(t) = \frac{1}{\alpha_k} \log \left[e^{\alpha v_{k0} - t/\tau_k} + \frac{V_{ki} G_k}{V_{kr}\sqrt{2}} h_k * G_{BR}(t) \right] \tag{20}$$

and using $V_{ko} = g_k V_{ki}$

$$V_{ko}(t) = V_{ki} \left[\frac{e^{\alpha_k v_{k0} - t/\tau_k}}{G_k} + \frac{V_{ki}}{V_{kr}\sqrt{2}} h_k * G_{BR} \right]^{-1} \tag{21}$$

where $\tau_k = 1/K_k \alpha_k V_{kr}$ and $h_k(t)$ has the same definition as $h_a(t)$ of (11) with $a \to k$.

The time response with two steps in G_{BR} for this KF when $G_k = 4$, $K_k = 100$, $\alpha_k = 1$, $v_{k0} = 0$ and $V_{kr} = 0.25$ is shown in Figure 6. From this, we see that V_{ko} decreases when G_{BR} increases which is exactly what we expect. This means that as the tune tracker PLL heads towards resonance (G_{BR} increases) the size of the kick V_{ko} decreases. As a check, we see that $V_{BR}/\sqrt{2} = V_{kr} = 0.25$ which is again exactly what we expect.

It is important to keep in mind that the KF regulates using the magnitude of the beam response G_{BR}. Therefore, in the real world implementation of KF, we have to calculate $G_{BR} = V_{BR}/V_{ko}$.

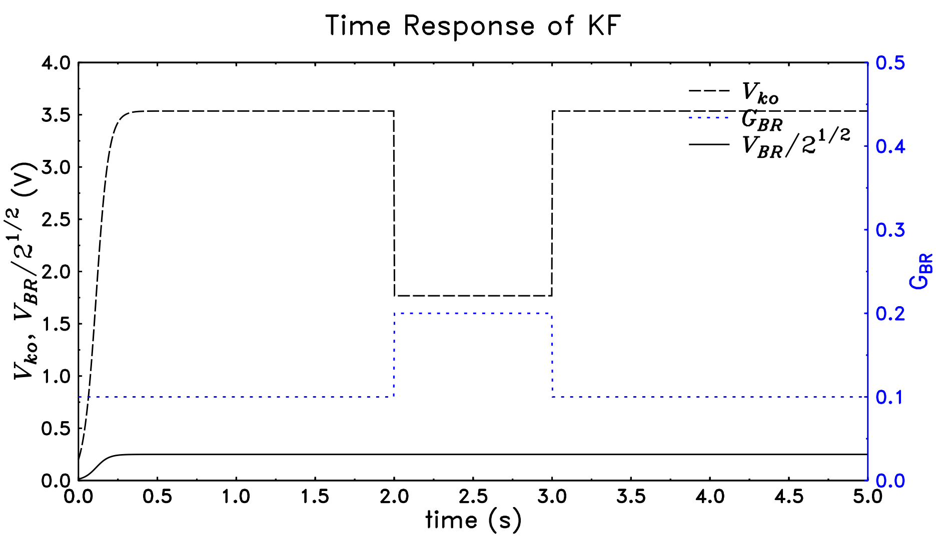

Figure 6 This is the time response of the KF when there are two steps in the beam response G_{BR}. We expect that V_{ko} decreases when G_{BR} increases and $V_{BR}/\sqrt{2} = V_{kr} = 0.25$ at steady state.

To combine the AGC and KF into one system and to ensure that they operate in such a manner as to not interfere or fight with each other, we must define carefully the working range for each of them.

For the AGC, its output voltage V_{ao} in response to the input voltage V_{ai} is to behave in the manner shown in Figure 7. In the real world, for sufficiently small signals say $V_{a1} < V_{ai}$, the AGC is unable to regulate because the VGA has insufficient gain. Therefore, the output V_{ao} scales linearly w.r.t. the input. For the input range $V_{a1} < V_{ai} < V_{a2}$, which is in the working range of the AGC, the output is regulated and approximately constant at $V_{ar}\sqrt{2}$ (if we use the rms detector). And finally, above V_{a2}, in some real world implementations, $V_{a2} = V_{ar}\sqrt{2}$ because VGAs cannot attenuate.

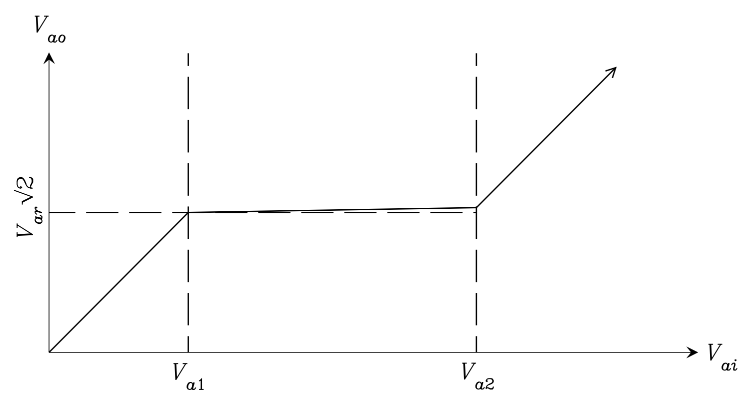

Figure 7 The response of the AGC that we have defined. The AGC does not regulate below V_{a1} and above V_{a2}.

For the KF response to work optimally with the AGC, its working range has to be defined. See Figure 8. For small beam responses $0 \leq G_{BR} < G_{BR1}$, the KF's kicker voltage V_{ko} will be as large as possible (and not blow up the beam emittance) so that the output signal V_{BR} is approximately constant. Above G_{BR1}, the KF stops regulating but keeps V_{ko} constant. G_{BR1} is chosen so that for $V_{ko} = V_{BR}/G_{BR1}$ there is no measurable emittance growth in this range and V_{BR} is within the working range of the AGC. For $G_{BR2} < G_{BR} < G_{BR3}$, the beam response has become large enough so that V_{ko} should be reduced to prevent emittance growth. For $G_{BR} > G_{BR3}$ or $V_{ko} = V_{BR}/G_{BR3}$, some real world implementations necessitate that the KF become non-regulatory because V_{ko} cannot be smaller than V_{ki}.

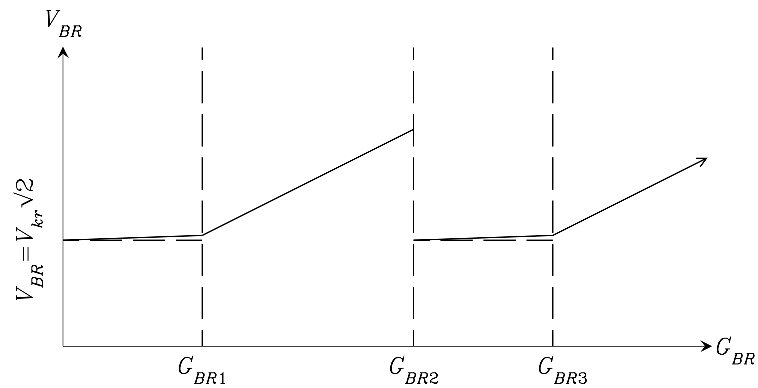

Figure 8 The response of a KF. The KF does not regulate between G_{BR1} and G_{BR2} and above G_{BR3}.

The combined AGC and KF circuit is shown in Figure 9. The AGC circuit is drawn in red and blue while the KF circuit is drawn in black and green. The position of the switches shown here have been set to the case when both G_{BR} and V_{BR} are in range. The green part of the KF circuit is used when G_{BR} is out of range and the blue part of the AGC circuit is used when V_{BR} is out of range. w_k is set to the most recent value of v_k and similarly w_a is also set to the most recent value of v_a.

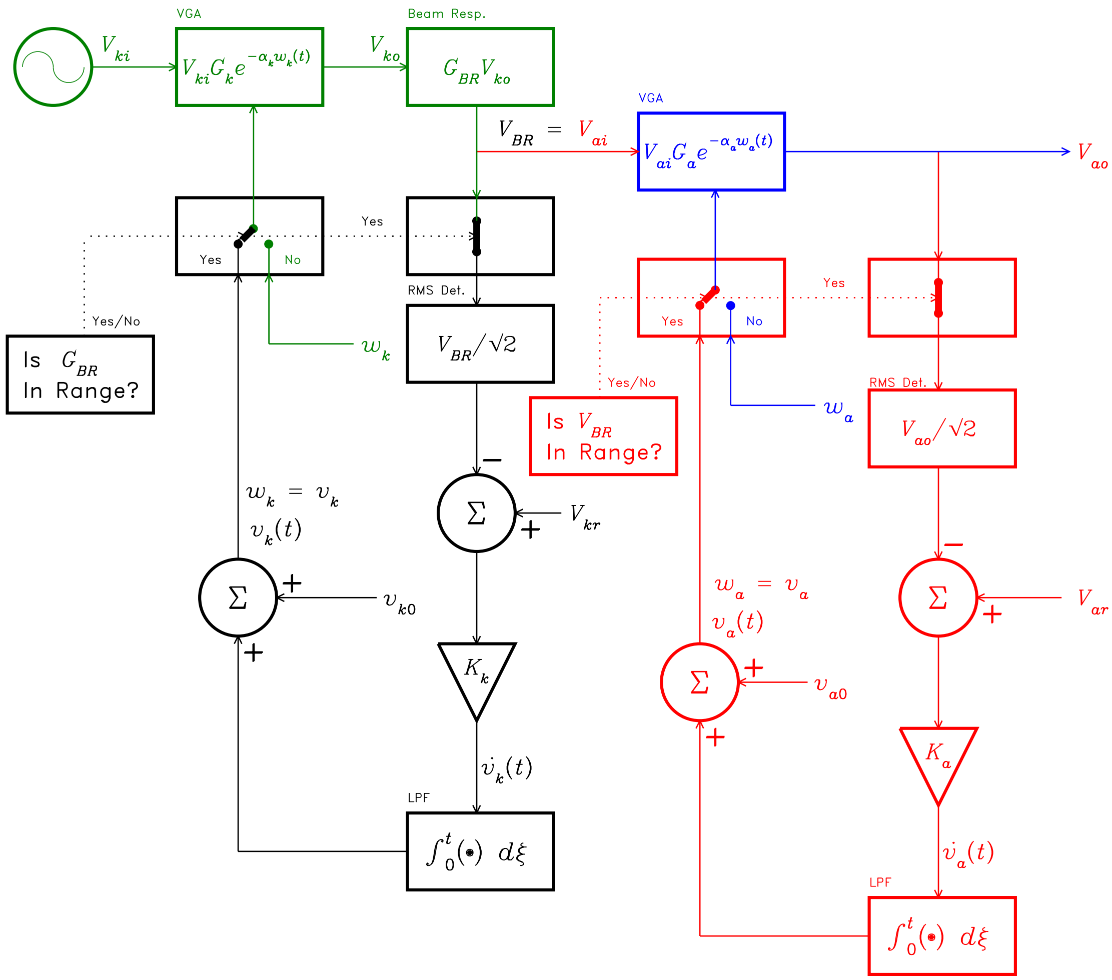

Figure 9 The combined AGC (red and blue) and KF (black and green) circuit. The position of the switches are for the case when both G_{BR} and V_{BR} are in range. The green part of the KF circuit is used when G_{BR} is out of range and the blue part of the AGC circuit is used when V_{BR} is out of range.

The Simulation

For the simulations of this circuit, we allow G_{BR} to ramp up and down. This mimics the situation where the tune tracker PLL approaches the beam resonance and then leaves it. The AGC and KF parameters are set to the values discussed in the *AGC* and *KF* sections plus the working range of the AGC has been set to $V_{a1} = 0.15$ V and $V_{a2} = 2.0$ V and the non-regulatory range of the KF has been set to between $G_{BR1} = 0.4$ and $G_{BR2} = 0.8$. The results of the simulation are shown in Figure 10. In this and subsequent figures, the working range of the AGC is the region shaded both in grey and cyan while the non-regulatory range of the KF is the cyan region.

We will discuss the simulation shown in Figure 10 by zooming into each part separately:

(*i*) **0.0 s to 0.2 s (Figure 11):** At the start of the simulation for a fixed G_{BR}, KF increases V_{ko} so that V_{BR} becomes larger. At 0.05 s, V_{BR} reaches the threshold where the AGC starts regulating and V_{ao} approaches a constant value once this threshold is crossed.

(*ii*) **0.5 s to 1.5 s (Figure 12):** As G_{BR} increases (the tune tracker PLL moves towards resonance), V_{ko} becomes smaller because it does not need to kick as hard to give the same V_{BR}. Once G_{BR} gets within the cyan band (above 1.15 s), the KF stops regulating and V_{ko} no longer increases but keeps a constant value. In this time interval, V_{ao} still remains constant because V_{BR} is within the working range of the AGC.

(*iii*) **1.5 s to 2.5 s (Figure 13):** As G_{BR} continues to increase (the tune tracker PLL gets close to resonance), G_{BR} gets outside the cyan band. Once outside the cyan band, the KF starts regulating and decreases the kick to the beam by lowering V_{ko}. During this time, the AGC is still working and keeps V_{ao} approximately constant.

(*iv*) **2.5 s to 5.0 s (Figure 14):** In this final simulation interval, as G_{BR} is reduced (the tune tracker PLL moving away from resonance), between 3.2 s and 3.7 s, the kick is set to a constant value because the KF stops regulation. Once G_{BR} leaves the cyan region above 3.7 s, the KF starts regulating and increases V_{ko} to keep V_{BR} within the AGC working range.

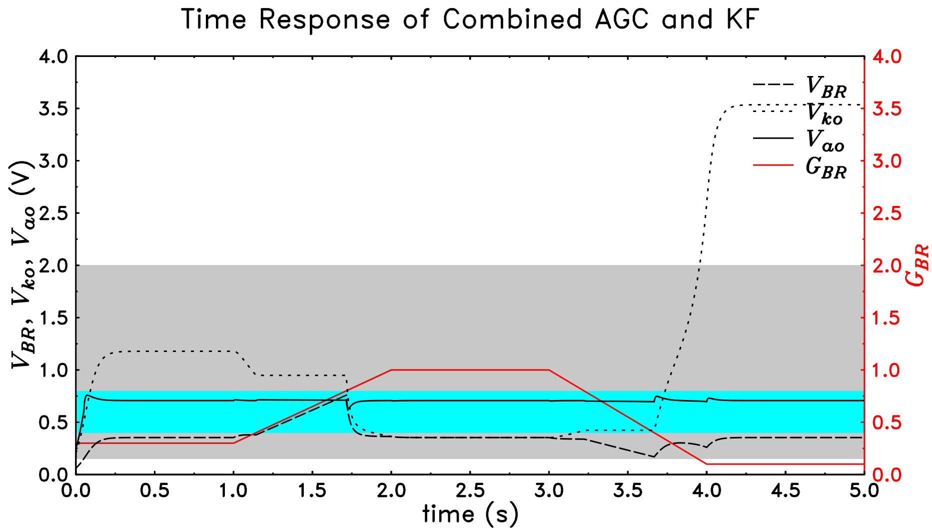

Figure 10 These are the overall results of the simulation described in the text. The band shaded both grey and cyan is the regulation range of the AGC. The area outside the cyan band is the regulation range of the KF.

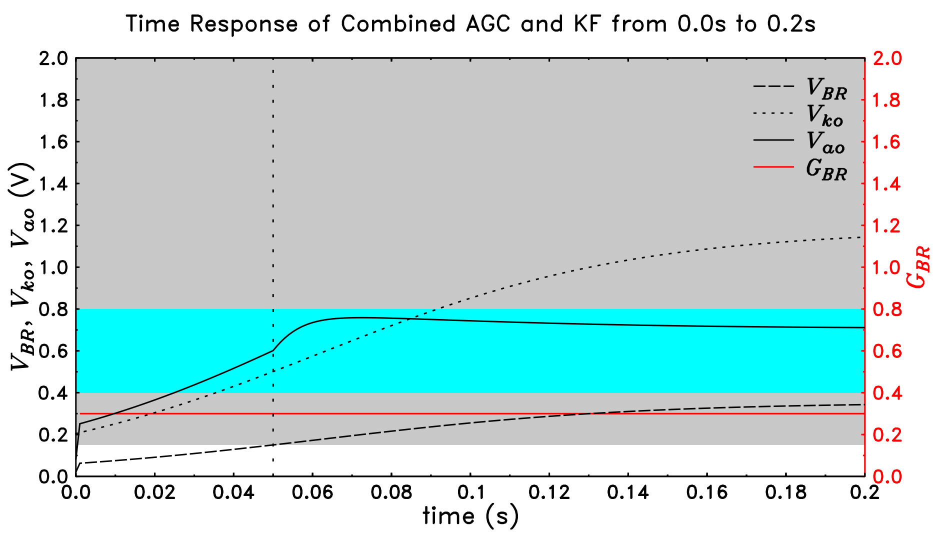

Figure 11 The first 0.2 s of the simulation.

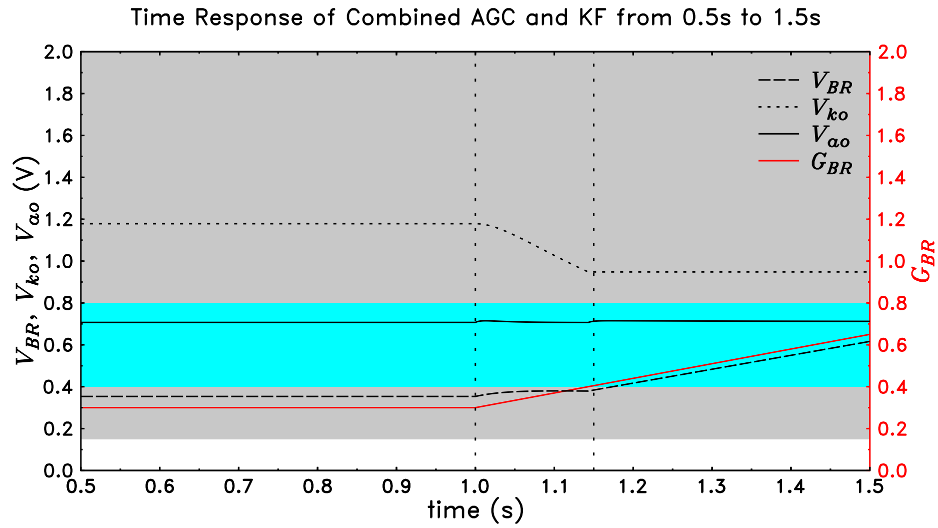

Figure 12 The simulation from 0.5 s to 1.5 s.

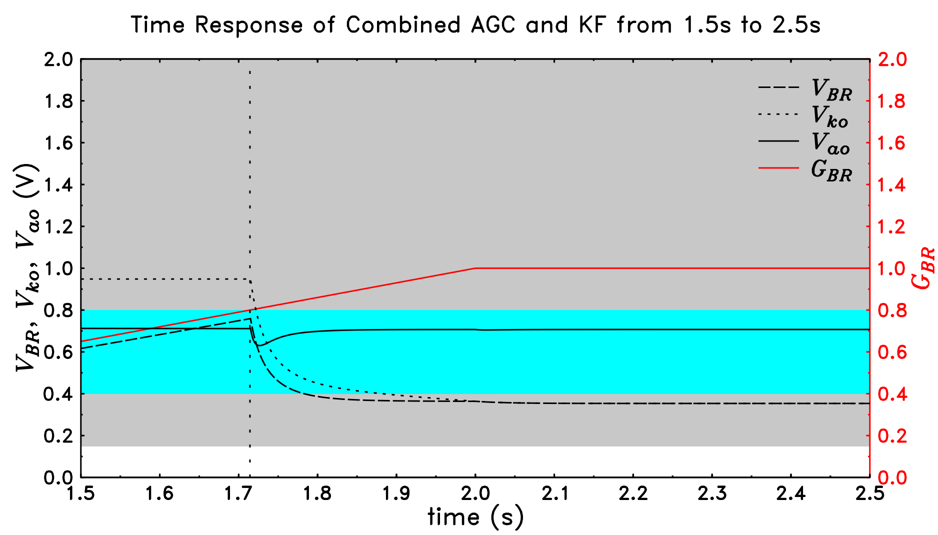

Figure 13 The simulation from 1.5 s to 2.5 s.

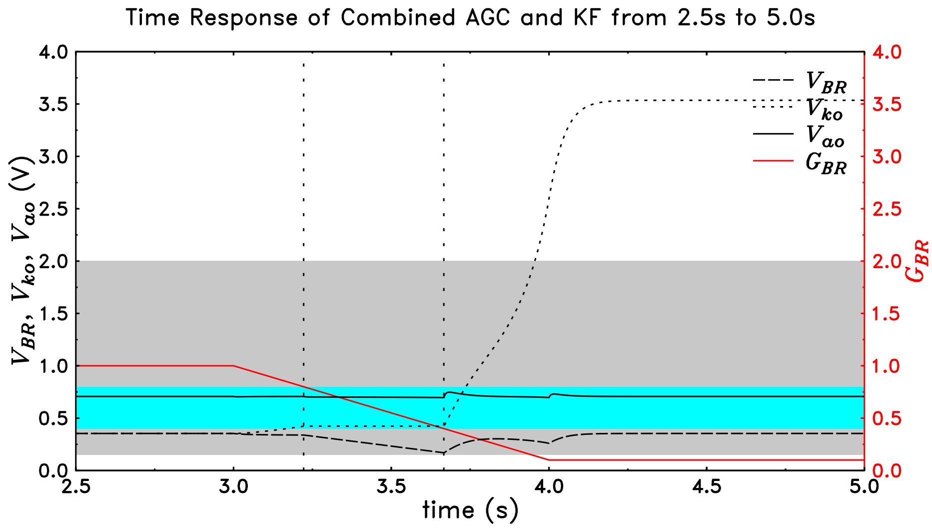

Figure 14 The simulation from 2.5 s to 5.0 s.

Settings Guide

When the combined AGC and KF circuits are used for the tune tracker PLL, we recommend that V_{ar}, V_{a1}, V_{a2}, V_{ki}, V_{kr}, G_{BR1}, and G_{BR2} be set to the following starting values. Machine studies should determine their final values. (Note that G_{BR3} is not a settable parameter because it comes from V_{BR} becoming so large that reducing V_{ko} to V_{ki} does not decrease V_{BR}.)

(*i*) $\boldsymbol{V_{ar}}$: The AGC reference voltage should be set so that $V_{ar}\sqrt{2}$ is approximately half the maximum input voltage of the downstream analogue to digital converter (ADC).

(*ii*) $\boldsymbol{V_{a1}}$: This should be set to about $V_{ar}/4$.

(*iii*) $\boldsymbol{V_{a2}}$: If the VGA cannot attenuate, then $V_{a2} = V_{ar}\sqrt{2}$.

(*iv*) $\boldsymbol{V_{kr}}$: When the KF is regulating, $V_{BR} = V_{kr}\sqrt{2}$. Therefore, V_{kr} should be set to between $V_{ar}/2$ to V_{ar}.

(*v*) $\boldsymbol{V_{ki}}$: If the VGA cannot attenuate, V_{ki} should be set so that when the beam is kicked at this level, the signal at the tune tracker detector is larger by about 6 dB.

(*vi*) $\boldsymbol{G_{BR1}}$: Set to about 6 dB above the base of the frequency response.

(*vii*) $\boldsymbol{G_{BR2}}$: Set to about 6 dB below the resonance of the frequency response.

When setting up the final values for G_{BR1} and G_{BR2}, we must take the care to not blow the emittance up.

CONCLUSION

We have shown how the AGC and KF can be combined so that they will work in synergy. The idea is to partition the working ranges of both the AGC and KF so that they do not compete with each other. A computer simulation of this combined loop for a changing beam response has been demonstrated plus analytic solutions have been derived for each individual loop.

REFERENCES

[1] C.Y. Tan, *The Tevatron Tune Tracker PLL — Theory, Implementation and Measurements*, TM-2275, Fermilab 2004.

[2] J.E. Ohlson, *Exact Dynamics of Automatic Gain Control* , IEEE Transactions on Communications, Jan 1974.