

# Impedance Effects in the PS2

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# Introduction

In this talk we give preliminary estimates of

- Microwave instability
- Transverse mode coupling instability
- Transverse coupled bunch instability
- Intrabeam scattering

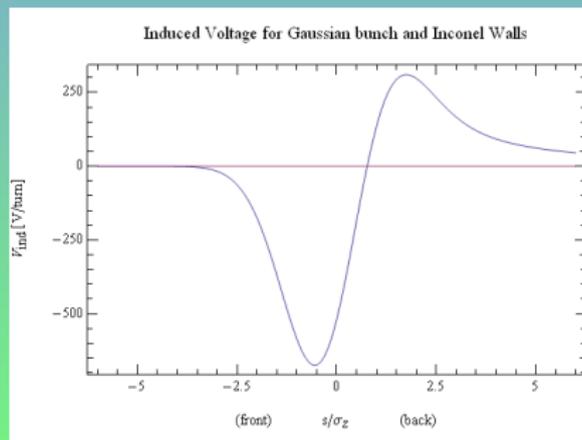
## Selected PS2 Parameters

Parameter	Value	Units
Circumference, $C$	1346.4	m
Chamber half apertures, $a_x$ by $a_y$	6 by 3.5	cm
Initial, final energies, $E_0, E_f$	4, 50	GeV
Bunch population, $N_b$	4.2	$10^{11}$
Average current, $I$	2.7	Amp
Long. emittance ( $4\pi\sigma_t\sigma_\delta$ ), $\epsilon_l$	0.6	eV-s
Norm. emittances $\gamma\epsilon_x = \gamma\epsilon_y$	3	$\pi \mu\text{m}$
Rms bunch length, $\sigma_{t0}, \sigma_{tf}$	3.8, 1	ns
Rms rel. energy spread, $\sigma_{\delta 0}, \sigma_{\delta f}$	3.2, 1	$10^{-3}$
Transition gamma, $\gamma_t$	35i	
Slippage factor, $\eta_0, \eta_f$	-0.037, -0.0012	
Synchrotron tune, $\nu_{s0}, \nu_{sf}$ ,	18, 0.8	$10^{-3}$
Vertical tune, $\nu_y$	8.2	
Average beta function, $\beta_y$	31	m

# Longitudinal Wake

- skin depth at (injection) bunch frequency:  $\delta = 88 \mu\text{m}$  (Inconel625),  $67 \mu\text{m}$  (SS316),  $16 \mu\text{m}$  (Al),  $10 \mu\text{m}$  (Cu)
- bunch wake scales as  $\sigma_z^{-3/2}$
- average voltage loss per turn, for Gaussian bunch:

$$\langle V_{ind} \rangle = -\frac{\Gamma(3/4)}{2^{5/2}\pi^2} \frac{ecN_b C}{b\sigma_z^{3/2}} \sqrt{\frac{Z_0}{\sigma_c}}$$



Voltage induced by a Gaussian bunch in the PS2, assuming Inconel walls;  $\langle V_{ind} \rangle = -309 \text{ V/turn}$ .

# Boussard Criterion for Microwave Instability

- Boussard criterion:

$$\frac{e\hat{l}|Z/n|}{2\pi|\eta|\mathcal{E}\sigma_\delta^2} \lesssim 1, \quad \text{or} \quad \frac{N_{th}}{N_b} \lesssim \frac{(2\pi)^{3/2}\sigma_z|\eta|\mathcal{E}\sigma_\delta^2}{e^2cN_b|Z/n|};$$

this criterion gives a very rough estimate of the threshold to the microwave instability.

- resistive wall impedance:

$$\frac{Z}{n} = (1 - i) \frac{\mathcal{C}}{2\pi b} \frac{1}{\delta_s \sigma_c} \frac{\omega_0}{\omega};$$

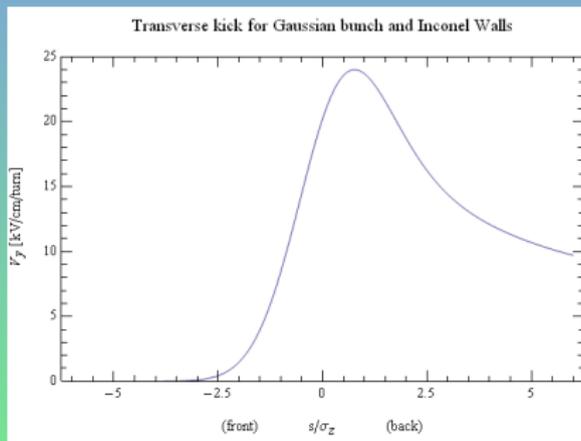
in Boussard criterion evaluate at  $\omega = c/\sigma_z$ .

- for PS2 with Inconel:  $|Z/n| = 0.67 \Omega$ ,  $N_{th}/N_b = 1900$ . (at injection), = 18. (at extraction).
- threshold at extraction is much lower because  $|\eta|\sigma_\delta^2$  is  $\sim 300$  times smaller at extraction than at injection.

## Transverse Wake

- bunch wake scales as  $\sigma_z^{-1/2}$
- average kick per cm offset per turn:

$$\langle V_y \rangle = \frac{\Gamma(1/4)}{2^{3/2}\pi^2} \frac{ecN_b C}{b^3 \sigma_z^{1/2}} \sqrt{\frac{Z_0}{\sigma_c}}$$



*Transverse kick induced by a Gaussian bunch in the PS2, assuming Inconel walls;  $\langle V_y \rangle = 17$  kV/cm/turn.*

# Transverse Mode Coupling Instability

- The threshold can be approximated by (see S. Krinsky, BNL 75019-2005-IR)

$$\frac{N_{th}}{N_b} \sim 0.7 \frac{4\pi\mathcal{E}\nu_s}{e^2 N_b \bar{\beta}_y k_y \mathcal{C}} ,$$

with the vertical kick factor  $k_y = \langle V_y \rangle / eN_b \mathcal{C}$ .

- for Inconel at injection,  $N_{th}/N_b = 12.$ , at extraction 6.6.

# Transverse Coupled Bunch Instability

- The main contribution to the growth rate of the coupled-bunch instability is normally the resistive wall impedance
- The growth rate of the fastest growing mode in a round, thick-walled pipe (see e.g. A. Wolski, et al, LBNL-59449, Feb. 2006):

$$\Gamma = \frac{c}{4\gamma} \frac{m_e I}{m_p I_A} \sqrt{\frac{C}{1 - [\nu_y]}} \langle \beta_y A_y \rangle$$

with

$$A_y = \frac{4}{\pi^{1/2} b^3} \sqrt{\frac{1}{Z_0 \sigma_c}}$$

- For Inconel at injection  $\Gamma = 4800 \text{ s}^{-1}$ , at extraction  $\Gamma = 470 \text{ s}^{-1}$ ; the growth times are equivalent to respectively 45., 475. turns

# Intrabeam Scattering (IBS)

- The growth in six-dimensional phase space due to local, two-particle scattering of protons is described by the IBS theory of Bjorken-Mtingwa (B-M).
- The growth rates are defined as

$$\frac{1}{T_x} = \frac{1}{\epsilon_x} \frac{d\epsilon_x}{dt} , \quad \frac{1}{T_y} = \frac{1}{\epsilon_y} \frac{d\epsilon_y}{dt} , \quad \frac{1}{T_p} = \frac{1}{\sigma_p} \frac{d\sigma_p}{dt} .$$

- We have solved B-M's formulas applied to the PS2 lattice.
- At injection  $[T_x, T_y, T_p] = [35, 55, -2000]$  min, at extraction they equal  $[30, -1.5 \times 10^4, 600]$  min.
- Note that, in the beam frame, at injection the (relative) longitudinal temperature,  $\sigma_p^2/\gamma^2 = 3.5 \times 10^{-7}$ , is significantly larger than the transverse temperature,  $\epsilon/\beta \approx 2 \times 10^{-8}$ .

# Conclusions

- We have given preliminary estimates of the importance of the microwave instability, the transverse mode coupling instability, the transverse coupled bunch instability, and the intrabeam scattering growth rates
- All threshold currents are much larger than the nominal current
- As a more accurate picture of the total impedance is obtained, and instability simulations are performed, more realistic threshold currents will be obtained
- For the transverse multi-bunch instability more accurate calculations of the low-frequency skin depth will be performed
- IBS growth times are long compared to the time the beam remains in the PS2 (1.2 s), so IBS can be ignored

# Proposed PS2 impedance tasks

## 2009

First estimates of single bunch instabilities

- microwave, transverse mode coupling instabilities
- evaluate space charge impedance
- intrabeam scattering growth rates

Resistive wall, multi-bunch transverse instability

## 2010

Build impedance model using best available data or from components of existing machines

- numerical calculation of impedance components, e.g. rf bellows, kickers, BPM's, transitions
- estimate single bunch growth rates and characteristics of instabilities

## 2011

Refine and iterate as time and funding permits

- write an impedance and instabilities section in the PS2 conceptual design report

### **Participants in this proposal**

SLAC: K. Bane, G. Stupakov, C. Ng

Effort: 0.5 FTE (FY09), 1.0 FTE (FY10), 0.5 FTE (FY11)

Travel: 2 person-weeks (FY09), 3 person-weeks (FY10), 2 person-weeks (FY11)