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Coupling correction at Tevatron using TBT data

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INTRODUCTION

The Fourier analysis of TBT data has been first applied at LEP in 1992 as a tool for measuring the uncoupled linear optics.

TBT data at the j^{th} BPM following a **single** kick in the z plane ($z \equiv x, y$)

$$z_n^j = \frac{1}{2} \sqrt{\beta_z^j} e^{i\Phi_z^j} A_z e^{iQ_z(\theta_j + 2\pi n)} + c.c.$$

with $n \equiv$ turn number $A_z = |A_z| e^{i\delta_z} \equiv$ constant of motion

$$\Phi_z \equiv \mu_z - Q_z \theta \quad (\text{periodic phase function})$$

Twiss functions:

$$\beta_z^j = |Z_j(Q_z)|^2 / A_z^2 \quad \mu_z^j = \arg(Z_j) - \delta_z$$

$$Z_j(Q_z) \equiv \text{Fourier component of } z_j$$

Amplitude fit:

$$|A_z|^2 = \frac{\sum_j 1/\beta_z^{0j}}{\sum_j 1/|Z_j(Q_z)|^2}$$

LINEAR COUPLING

Method of the **variation of constants**:

The general solution of the perturbed motion keeps the form of the unperturbed one with constants depending on time^a

Hamiltonian in presence of a perturbation, H_1 ,

$$H = [H_0 + H_1](\vec{z}) = [U_0 + U_1](\vec{A})$$

Equations of motion

$$\frac{dA_j}{dt} = \sum_m [A_j, A_m] \frac{\partial U_1}{\partial A_m}$$

When the unperturbed Hamiltonian describe the **betatron motion**, thus

$$\frac{dA_z}{d\theta} = i \frac{\partial U_1}{\partial A_z^*} \quad \frac{dA_z^*}{d\theta} = -i \frac{\partial U_1}{\partial A_z}$$

^a θ or s in our case

For perturbation fields generating **linear coupling** (Guignard)

$$U_1(\vec{a}) = \frac{1}{2} [C_+(\theta) a_x a_y + C_+^*(\theta) a_x^* a_y^* + C_-(\theta) a_x a_y^* + C_-^*(\theta) a_x^* a_y]$$
$$a_z \equiv A_z e^{iQ_z \theta}$$

where

$$C_{\pm}(\theta) \equiv \frac{R\sqrt{\beta_x\beta_y}}{2B\rho} \left\{ \left(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y} \right) + B_\theta \left[\left(\frac{\alpha_x}{\beta_x} - \frac{\alpha_y}{\beta_y} \right) - i \left(\frac{1}{\beta_x} \mp \frac{1}{\beta_y} \right) \right] \right\} e^{i(\Phi_x \pm \Phi_y)}$$

and

$$\Phi_z \equiv \mu_z - Q_z \theta$$

“Ansatz” (Yuri Alexahin)

$$a_x(\theta) = a_{x0}(\theta) + w_-^*(\theta)a_{y0}(\theta) + w_+^*(\theta)a_{y0}^*(\theta)$$

$$a_y(\theta) = a_{y0}(\theta) - w_-(\theta)a_{x0}^*(\theta) + w_+^*(\theta)a_{x0}^*(\theta)$$

Inserting into the equation of motion and keeping 1th order terms one finds the equations for w_{\pm}

$$2ie^{-iQ_{\pm}\theta} \frac{d}{d\theta} e^{iQ_{\pm}\theta} w_{\pm}(\theta) = C_{\pm}(\theta)$$

which **periodic** solutions are

$$w_{\pm}(\theta) = - \int_0^{2\pi} d\theta' \frac{C_{\pm}(\theta')}{4 \sin \pi Q_{\pm}} e^{-iQ_{\pm}[\theta-\theta' - \pi \text{sign}(\theta-\theta')]}$$

with

$$Q_{\pm} \equiv Q_x \pm Q_y$$

The functions $\tilde{w}_{\pm} \equiv w_{\pm} e^{iQ_{\pm}\theta}$ are

- constant in coupler free regions
- experience a discontinuity $-iC_{\pm}\ell/2R$ at coupler locations
- are constant on the resonances $Q_x \pm Q_y = int.$

Minimum tune split

$$\Delta \equiv |\bar{C}_-^{n_-}| \quad \bar{C}_{\pm}^{n_{\pm}} = \frac{1}{2\pi} \int_0^{2\pi} d\theta C_{\pm} e^{in_{\pm}\theta} = \frac{n_{\pm} - Q_{\pm}}{\pi} \int_0^{2\pi} d\theta w_{\pm} e^{in_{\pm}\theta}$$

with

$$n_{\pm} \equiv \text{Round}(Q_x \pm Q_y)$$

Linear coupling computation through TBT analysis

TBT beam position at the j^{th} vertical BPM following a horizontal kick

$$y_n^j = \left[\sqrt{\beta_y^j} \left(e^{-i\Phi_y^j} w_+^j - e^{i\Phi_y^j} w_-^j \right) - \sqrt{\beta_x^j} e^{i\Phi_x^j} \sin \chi_j \right] A_x e^{iQ_x(\theta_j + 2\pi n)} + c.c.$$

TBT beam position at the j -th horizontal BPM following a vertical kick

$$x_n^j = \left[\sqrt{\beta_x^j} \left(e^{-i\Phi_x^j} w_+^j + e^{i\Phi_x^j} w_-^{*j} \right) + \sqrt{\beta_y^j} e^{i\Phi_y^j} \sin \chi_j \right] A_y e^{iQ_y(\theta_j + 2\pi n)} + c.c.$$

($\chi_j \equiv$ tilt of the j^{th} BPM).

The FFT of y^j at Q_x , $Y^j(Q_x)$, for a horizontal kick ($X^j(Q_y)$ for a vertical one) is proportional to the coupling functions $w_{\pm}(\theta_j)$.

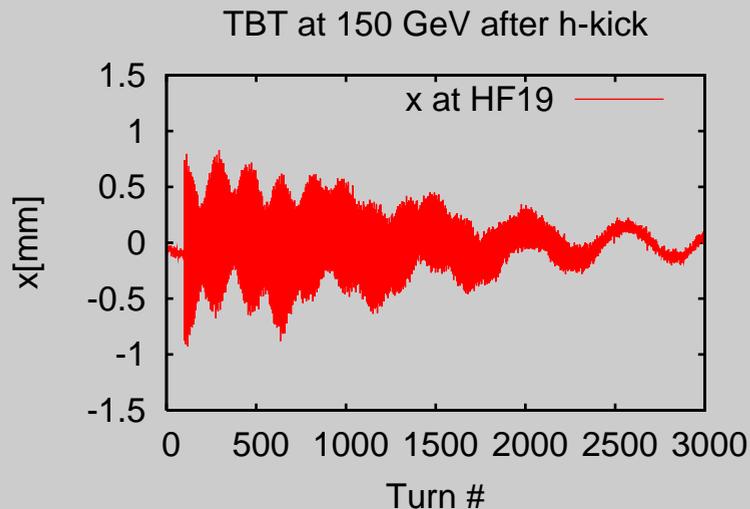
Assuming χ_j known, we get 2 real equations per BPM in 4 unknowns. When between two consecutive monitors there are no strong source of coupling, the four equations can be solved in favor of $w_{\pm}(\theta_j) = w_{\pm}(\theta_{j+1})$.

TEVATRON EXAMPLES

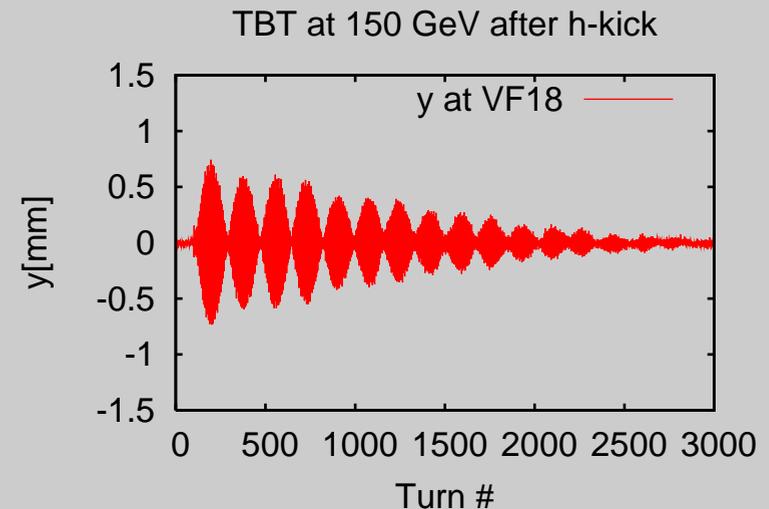
TEVATRON has 118 horizontal and 118 vertical BPM's. They can store 8192 positions data per BPM. The upgrade of their electronics allows a precise measurement of the TBT beam position (resolution $\simeq 50 \mu\text{m}$) and made possible the use of TBT techniques.

Under "ideal" conditions the oscillations following a kick last some thousand turns

TBT position after a horizontal kick

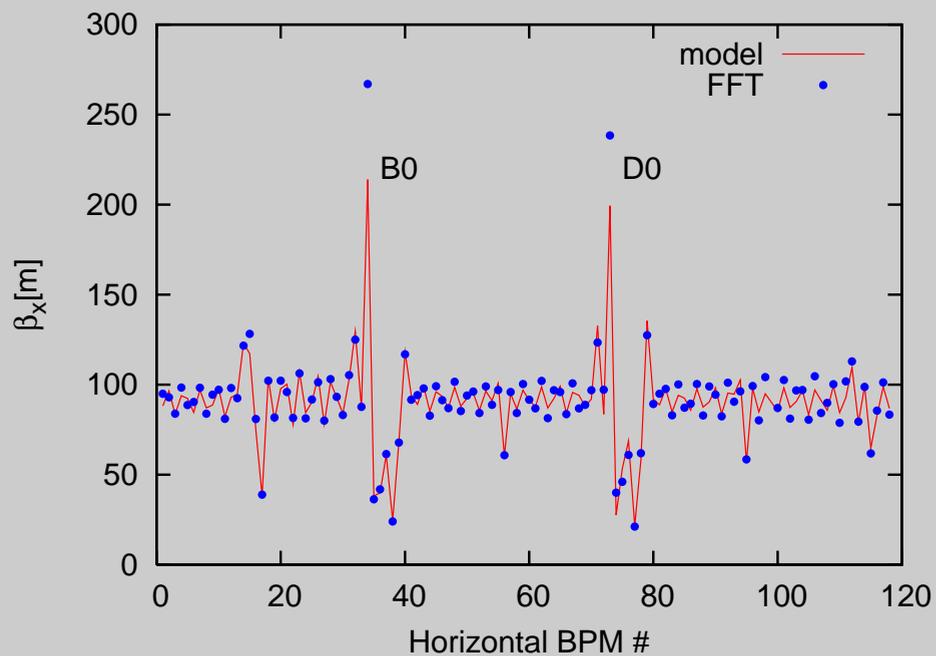


TBT position at HF19

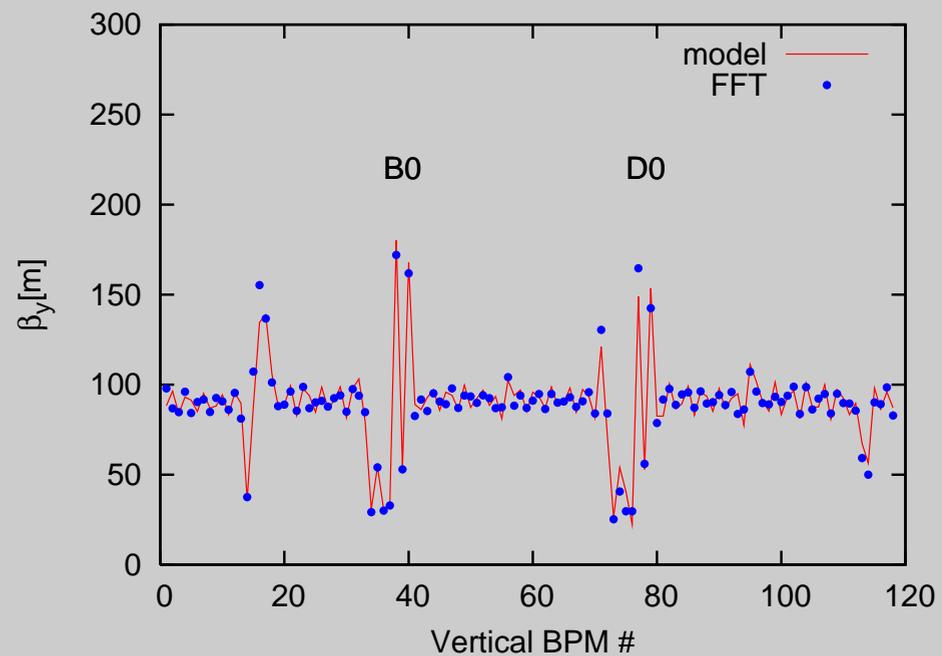


TBT position at VF18

Reconstructed Injection Optics (November 2005 data)

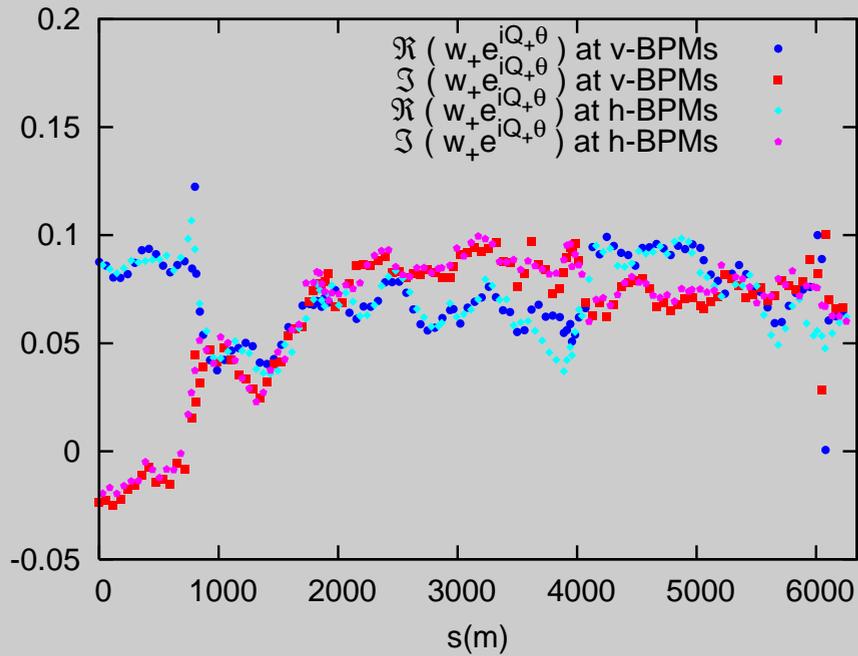


Horizontal

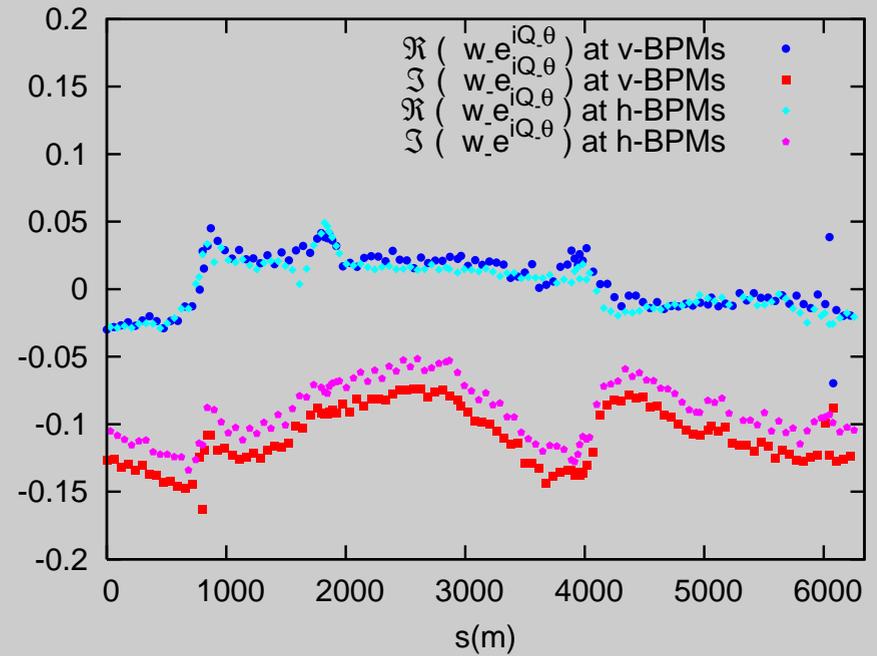


Vertical

Coupling functions (November 2005 data)



\tilde{w}^+



\tilde{w}^-

Jumps visible around 1000 (SQA0), 1500 (A38) and 4000 (D16) meters.

EFFECT OF ERRORS

BPM's **calibration errors** affect the value of β_z^j computed through the Fourier analysis.

The effect of **random** calibration errors results in a unphysical beta-beating which is likely to average away when computing the oscillation amplitude.

A **systematic** calibration error has no effect on the evaluation of the β functions, but results in a wrong estimate of the oscillation amplitude and therefore of w_{\pm}^j (unless the error is the same, for both horizontal and vertical BPM's).

By requiring

$$M_{12}^{meas} = M_{12}^{theo} \quad (\text{or } M_{34}^{meas} = M_{34}^{theo})$$

one can compute β_z^j resorting only on the measured phase advance. This requires (at least) three (consecutive) BPM's. Comparison with the value computed through the Fourier analysis may be used to calibrate the BPM's involved. However this will *not* correct for a possible systematic calibration error.

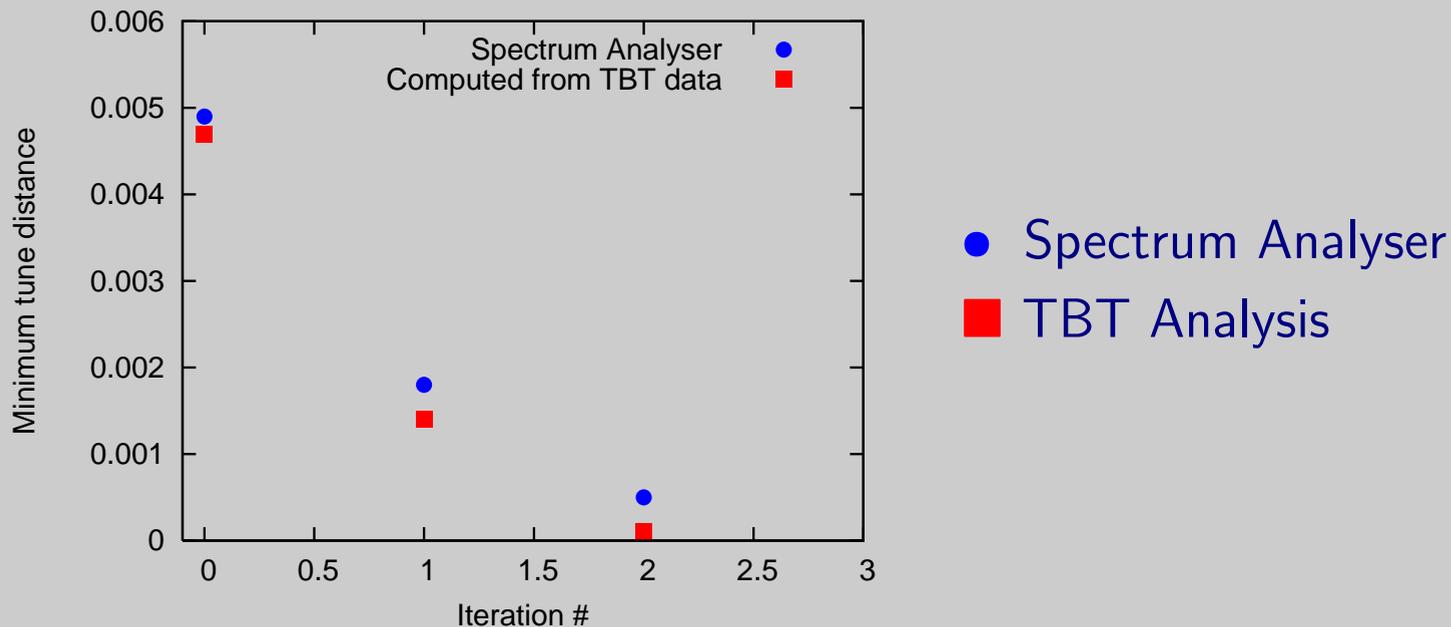
Through simulations^a we have estimated that the error on the evaluation of $|\bar{C}_-|$ is

- $\sim 2.5\%$ for 5% systematic calibration error of either horizontal or vertical BPM's (they cancel out when the error has the *same* value)
- 0.5% for 5% random calibration errors
- a systematic tilt by 1° of all BPM's results in a error $|\delta\bar{C}_-| \simeq 0.0002$
- the error due to random tilts is negligible.

^a 50 μm resolution assumed

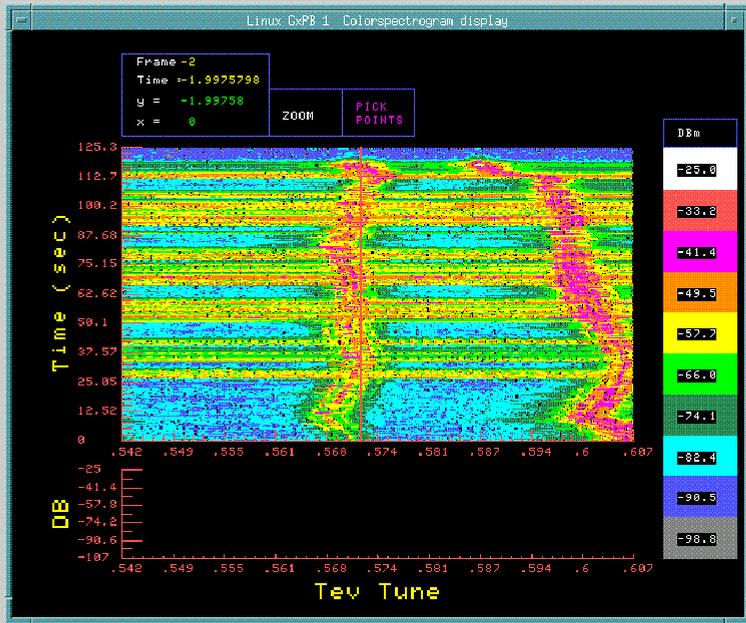
An application program for the TBT analysis has been integrated in the TEVATRON control system and is used **routinely** at **shot set up** for correcting the minimum tune split $\Delta \equiv |\bar{C}_-|$ with two skew quadrupole circuits. The correction is accurate and fast compared to the usual method of finding the minimum tune split by empirical adjustment of the skew quadrupole circuits.

Minimum tune split measured with S.A. and computed from TBT data

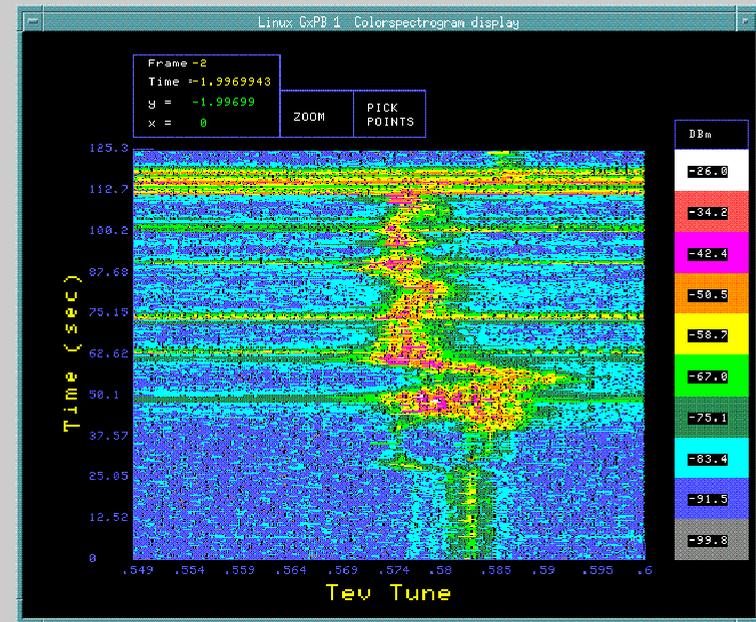


TEVATRON being a **fast ramping** machine (83 seconds from 150 to 980 GeV), the TBT analysis is a very practical method for measuring optics and coupling also during **acceleration**. The application has been used several times, in particular after long shut-downs, for decoupling on the ramp.

Tunes on ramp after 2006 shut-down



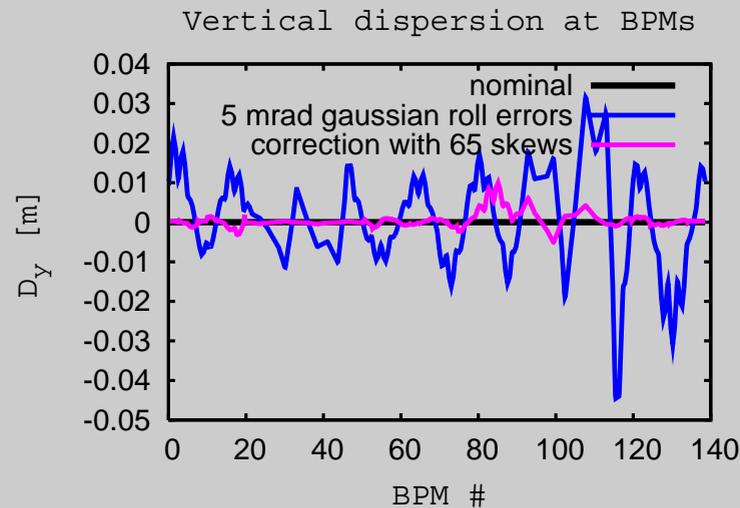
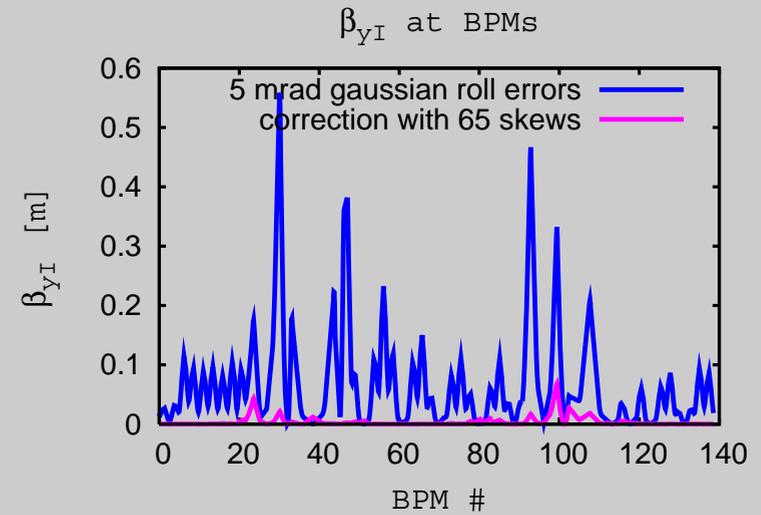
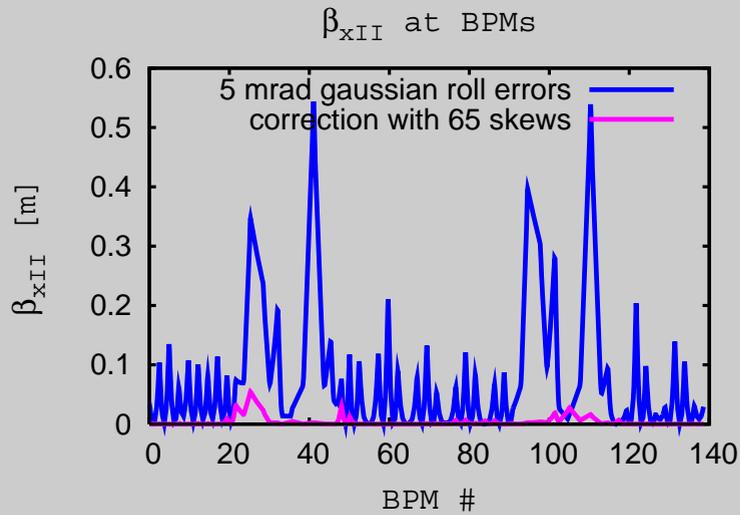
Very first ramp before correction



...after correction

SIMULATION FOR KEK ATF

Distributed simultaneous correction of w^\pm & spurious vertical dispersion



FIT OF TBT DATA USING MAD-X

The Fourier analysis of the measured TBT data

$$\begin{aligned}x_n &= A_I \sqrt{\beta_{xI}} \cos(\phi_{xI} + \delta_I + 2\pi n Q_I) + \\ &\quad A_{II} \sqrt{\beta_{xII}} \cos(\phi_{xII} + \delta_{II} + 2\pi n Q_{II}) \\ y_n &= A_I \sqrt{\beta_{yI}} \cos(\phi_{yI} + \delta_I + 2\pi n Q_I) + \\ &\quad A_{II} \sqrt{\beta_{yII}} \cos(\phi_{yII} + \delta_{II} + 2\pi n Q_{II})\end{aligned}$$

gives the coupled **Mais-Ripken** twiss functions $\beta_{zI,II}$ and $\phi_{zI,II}$ ($z \equiv x, y$), a part for the constants of motion $A_{I,II}$ and $\delta_{I,II}$.

The **eigenvectors** of the coupled transport matrix are related to the Mais-Ripken twiss functions

$$\begin{aligned} V_{11} &\equiv \sqrt{\beta_{xI}} \cos \phi_{xI} & V_{12} &\equiv \sqrt{\beta_{xI}} \sin \phi_{xI} \\ V_{13} &\equiv \sqrt{\beta_{xII}} \cos \phi_{xII} & V_{14} &\equiv \sqrt{\beta_{xII}} \sin \phi_{xII} \\ V_{31} &\equiv \sqrt{\beta_{yI}} \cos \phi_{yI} & V_{32} &\equiv \sqrt{\beta_{yI}} \sin \phi_{yI} \\ V_{33} &\equiv \sqrt{\beta_{yII}} \cos \phi_{yII} & V_{34} &\equiv \sqrt{\beta_{yII}} \sin \phi_{yII} \end{aligned}$$

Taking into account that the BPMs may have (unknown) calibration errors and may be tilted around the longitudinal axis^a the actual eigenvector components are related to the measured ones, \bar{V}_{lk}^i ($i \equiv$ BPM index), by

$$\begin{aligned} \frac{1}{A_I} [\cos(\delta_I) \bar{V}_{11}^i + \bar{V}_{12}^i \sin(\delta_I)] &= \frac{1}{r_i} V_{11}^i + \frac{\chi_i}{r_i} V_{31}^i \\ \frac{1}{A_I} [-\sin(\delta_I) \bar{V}_{11}^i + \bar{V}_{12}^i \cos(\delta_I)] &= \frac{1}{r_i} V_{12}^i + \frac{\chi_i}{r_i} V_{32}^i \\ \frac{1}{A_{II}} [\cos(\delta_{II}) \bar{V}_{13}^i + \bar{V}_{14}^i \sin(\delta_{II})] &= \frac{1}{r_i} V_{13}^i + \frac{\chi_i}{r_i} V_{33}^i \\ \frac{1}{A_{II}} [-\sin(\delta_{II}) \bar{V}_{13}^i + \bar{V}_{14}^i \cos(\delta_{II})] &= \frac{1}{r_i} V_{14}^i + \frac{\chi_i}{r_i} V_{34}^i \\ \frac{1}{A_I} [\cos(\delta_I) \bar{V}_{31}^i + \bar{V}_{32}^i \sin(\delta_I)] &= \frac{1}{r_i} V_{31}^i - \frac{\chi_i}{r_i} V_{11}^i \\ \frac{1}{A_I} [-\sin(\delta_I) \bar{V}_{31}^i + \bar{V}_{32}^i \cos(\delta_I)] &= \frac{1}{r_i} V_{32}^i - \frac{\chi_i}{r_i} V_{12}^i \\ \frac{1}{A_{II}} [\cos(\delta_{II}) \bar{V}_{33}^i + \bar{V}_{34}^i \sin(\delta_{II})] &= \frac{1}{r_i} V_{33}^i - \frac{\chi_i}{r_i} V_{13}^i \\ \frac{1}{A_{II}} [-\sin(\delta_{II}) \bar{V}_{33}^i + \bar{V}_{34}^i \cos(\delta_{II})] &= \frac{1}{r_i} V_{34}^i - \frac{\chi_i}{r_i} V_{14}^i \end{aligned}$$

^aThe BPM reading is related to the actual beam position by

$$x^{meas} = \frac{x + y \tan \chi}{r_x} \quad y^{meas} = \frac{y - x \tan \chi}{r_y}$$

with $\chi \equiv$ BPM tilt and $r_z \equiv z/z^{meas}$ ($z \equiv x, y$).

Goal: adjust

- quadrupole **gradient** and **tilt**
- BPMs **calibration** and **tilt**
- $A_{I,II}$ and $\delta_{I,II}$

in order to fit the measured eigenvector values at the BPMs.

It could be a good alternative to time consuming (for both data taking and computation) Orbit Response Matrix methods currently used.

MAD-X is capable of matching *coupled* optics and allows *user-defined* expressions in matching constraints (“macros”).

MAD-X **TWISS** uses Edwards-Teng formalism.

MAD-X **PTC_TWISS** uses Mais-Ripken formalism, but it is too slow for matching purposes.

The two formalism are of course related, the relationships between the two sets of twiss functions being

$$\begin{aligned}\beta_{xI} &= \kappa\beta_1 & \beta_{yII} &= \kappa\beta_2 & \phi_{xI} &= \varphi_1 & \phi_{yII} &= \varphi_2 \\ \beta_{xII} &= \kappa[R_{22}(R_{22}\beta_2 + 2R_{12}\alpha_2) + R_{12}^2\gamma_2] \\ \beta_{yI} &= \kappa[R_{11}(R_{11}\beta_1 - 2R_{12}\alpha_1) + R_{12}^2\gamma_1] \\ \phi_{xII} &= \varphi_2 - \arctan[R_{12}/(R_{22}\beta_2 + R_{12}\alpha_2)] \\ \phi_{yI} &= \varphi_1 + \arctan[R_{12}/(R_{11}\beta_1 - R_{12}\alpha_1)]\end{aligned}$$

with $\kappa \equiv 1/(1 + |\mathbf{R}|)$, \mathbf{R} being a 2×2 matrix, also computed by MAD-X.

Use MAD-X macros to define

- Mais-Ripken functions in terms of Edwards-Teng ones
- constraints & variables

Application to Tevatron

- Number of observation points: 2×118
- Current Tevatron model (A.Valishev): 216 normal and 216 skew thin quadrupoles to simulate gradient and tilt errors. We must add the unknown BPM calibrations and tilts, with the additional condition $\langle r_i \rangle = 1$, and the oscillation amplitude and phase.

All together: 908 parameters and 945 constraints. The fit is time consuming, therefore

- simplify input by **lumping** elements and declare them as one MATRIX
 - **split** fit
 - use MAD-X for fitting magnetic elements
 - fit linearly BPMs calibration and tilt by a different code
- and iterate

But

- Still too slow for a console application → write our own minimisation code.
On the way.
- No convergence by fitting real data (July 07): mismatch between some definitions and MAD-X implementation???

Acknowledgements

Linear Coupling: [Yuri Alexahin](#)

MAD-X fit: [Yuri Alexahin](#), Valery Kapin, Frank Schmidt, Sergey Senkin