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TBT data analysis for optics diagnostic

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INTRODUCTION

The Fourier analysis of TBT data has been first applied at LEP in 1992 as a tool for measuring the uncoupled linear optics.

TBT data at the j^{th} BPM following a single kick in the z plane $(z \equiv x, y)$

$$z_n^j = \frac{1}{2}\sqrt{\beta_z^j}\mathrm{e}^{i\Phi_z^j} A_z \mathrm{e}^{iQ_z(\theta_j + 2\pi n)} + c.c.$$

with $\quad n \equiv$ turn number $\quad A_z = |A_z|\mathrm{e}^{i\delta_z} \equiv$ constant of motion

$\Phi_z \equiv \mu_z - Q_z\theta \quad$ (periodic phase function)

Twiss functions:

$$\beta_z^j = |Z_j(Q_z)|^2/A_z^2 \qquad \mu_z^j = \arg\,(Z_j) - \delta_z$$

$Z_j(Q_z) \equiv$ Fourier component of z_j

Amplitude fit:

$$|A_z|^2 = \frac{\sum_j 1/\beta_z^{0j}}{\sum_j 1/|Z_j(Q_z))|^2}$$

LINEAR COUPLING

Method of the variation of constants:

The general solution of the perturbed motion keeps the form of the unperturbed one with constants depending on time[a]"

Hamiltonian in presence of a perturbation, H_1,

$$U = U_0 + H_1(\vec{z}) = U_0 + H_1(V\vec{A}) = U_0 + U_1(\vec{A})$$

Equations of motion

$$\frac{dA_j}{dt} = \Sigma_m [A_j, A_m] \frac{\partial U_1}{\partial A_m}$$

When the unperturbed Hamiltonian describe the betatron motion, thus

$$\frac{dA_z}{d\theta} = i \frac{\partial U_1}{\partial A_z^*} \qquad \frac{dA_z^*}{d\theta} = -i \frac{\partial U_1}{\partial A_z}$$

[a] θ in our case

For perturbation fields generating linear coupling (Guignard)

$$U_1(\vec{a}) = \frac{1}{2}[C_+(\theta)a_x a_y + C_+^*(\theta)a_x^* a_y^* + C_- a_x a_y^* + C_-^* a_x^* a_y]$$

$$a_z \equiv A_z e^{iQ_z\theta}$$

where

$$C_\pm(\theta) \equiv \frac{R\sqrt{\beta_x\beta_y}}{2B\rho}\Bigg\{\left(\frac{\partial B_x}{\partial x} - \frac{\partial B_y}{\partial y}\right) + B_\theta\left[\left(\frac{\alpha_x}{\beta_x} - \frac{\alpha_y}{\beta_y}\right) - i\left(\frac{1}{\beta_x} \mp \frac{1}{\beta_y}\right)\right]\Bigg\}\mathrm{e}^{i(\Phi_x \pm \Phi_y)}$$

and

$$\Phi_z \equiv \mu_z - Q_z\theta$$

"Ansatz" (Yuri Alexahin)

$$a_x(\theta) = a_{x0}(\theta) + w_-^*(\theta) a_{y0}(\theta) + w_+^*(\theta) a_{y0}^*(\theta)$$

$$a_y(\theta) = a_{y0}(\theta) - w_-(\theta) a_{x0}^*(\theta) + w_+^*(\theta) a_{x0}^*(\theta)$$

Inserting into the equation of motion and keeping the first order terms one finds the equations for $w_\pm$

$$2i\mathrm{e}^{-iQ_\pm\theta} \frac{d}{d\theta} \mathrm{e}^{iQ_\pm\theta} w_\pm(\theta) = C_\pm(\theta)$$

which periodic solutions are

$$w_\pm(\theta) = -\int_0^{2\pi} d\theta' \frac{C_\pm(\theta')}{4\sin\pi Q_\pm} \mathrm{e}^{-iQ_\pm[\theta-\theta'-\pi\,\mathrm{sign}(\theta-\theta')]}$$

with

$$Q_\pm \equiv Q_x \pm Q_y$$

The functions $\tilde{w}_\pm \equiv w_\pm \mathrm{e}^{iQ_\pm \theta}$ are

- constant in coupler free regions
- experience a discontinuity $-iC_\pm \ell / 2R$ at coupler locations
- on the resonances $Q_x \pm Q_y = int$ are constant.

Minimum tune split

$$\Delta \equiv |\bar{C}_-^{n_-}| \qquad \bar{C}_\pm^{n_\pm} = \frac{1}{2\pi} \int_0^{2\pi} d\theta \, C_\pm \mathrm{e}^{in_\pm \theta} = \frac{n_\pm - Q_\pm}{\pi} \int_0^{2\pi} d\theta \, w_\pm \mathrm{e}^{in_\pm \theta}$$

with

$$n_\pm \equiv \mathrm{Round}(Q_x \pm Q_y)$$

Linear coupling computation through TBT analysis

TBT beam position at the j$^{\text{th}}$ vertical BPM following a horizontal kick

$$y_n^j = \left[\sqrt{\beta_y^j}\left(\mathrm{e}^{-i\Phi_y^j} w_+^j - \mathrm{e}^{i\Phi_y^j} w_-^j\right) - \sqrt{\beta_x^j}\mathrm{e}^{i\Phi_x^j} \sin\chi_j\right] A_x \mathrm{e}^{iQ_x(\theta_j + 2\pi n)} + c.c.$$

TBT beam position at the j-th horizontal BPM following a vertical kick

$$x_n^j = \left[\sqrt{\beta_x^j}\left(\mathrm{e}^{-i\Phi_x^j} w_+^j + \mathrm{e}^{i\Phi_x^j} w_-^{*j}\right) + \sqrt{\beta_y^j}\mathrm{e}^{i\Phi_y^j} \sin\chi_j\right] A_y \mathrm{e}^{iQ_y(\theta_j + 2\pi n)} + c.c.$$

($\chi_j \equiv$ tilt of the j$^{\text{th}}$ BPM).

The FFT of y^j at Q_x, $Y^j(Q_x)$, for a horizontal kick ($X^j(Q_y)$ for a vertical one) is proportional to the *coupling functions* $w_\pm(\theta_j)$.

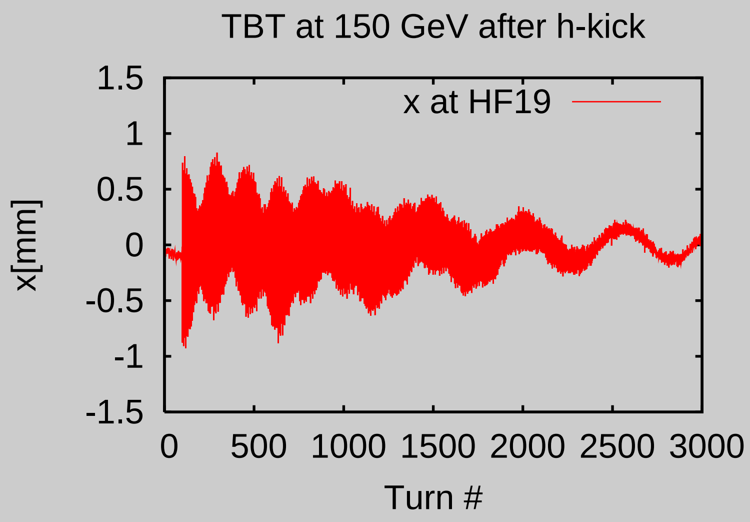
Assuming χ_j known, we get two real equations per BPM in 4 unknowns. When between two consecutive monitors there are no strong source of coupling, the four equations can be solved in favor of $w_\pm(\theta_j) = w_\pm(\theta_{j+1})$.

TEVATRON EXAMPLES

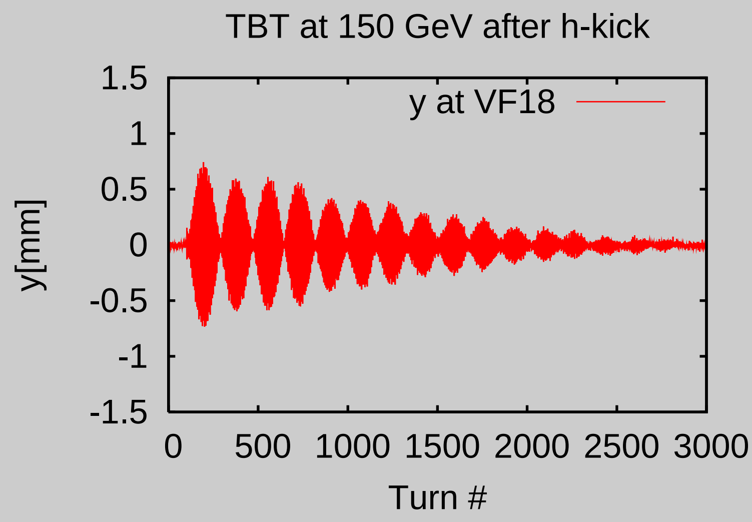
TEVATRON has 118 horizontal and 118 vertical BPM's. They can store 8192 positions data per BPM. The upgrade of their electronics allows a precise measurement of the TBT beam position (resolution $\simeq 50\ \mu$m) and made possible the use of TBT techniques.

Under "ideal" conditions the oscillations following a kick last some thousand turns

TBT position after a horizontal kick

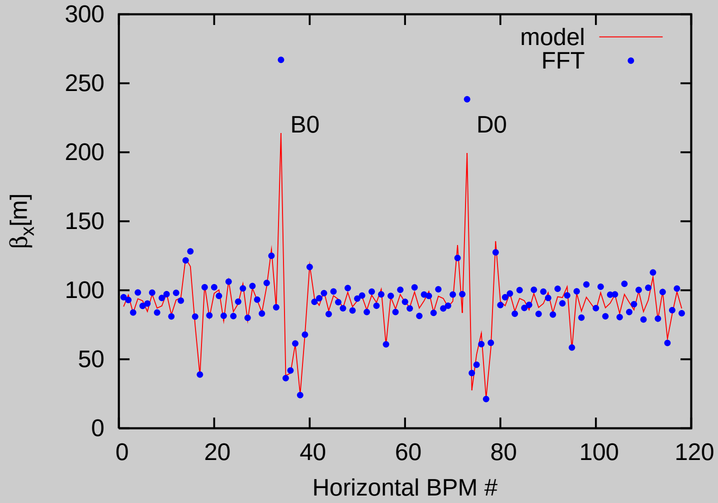


TBT position at HF19

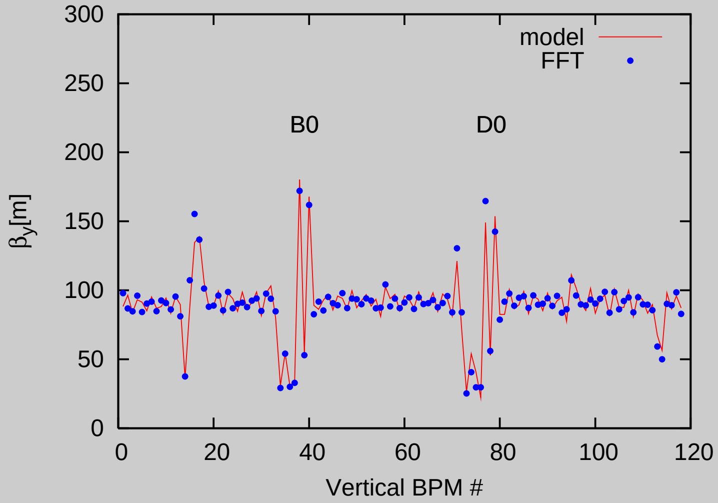


TBT position at VF18

Reconstructed Injection Optics (2006)

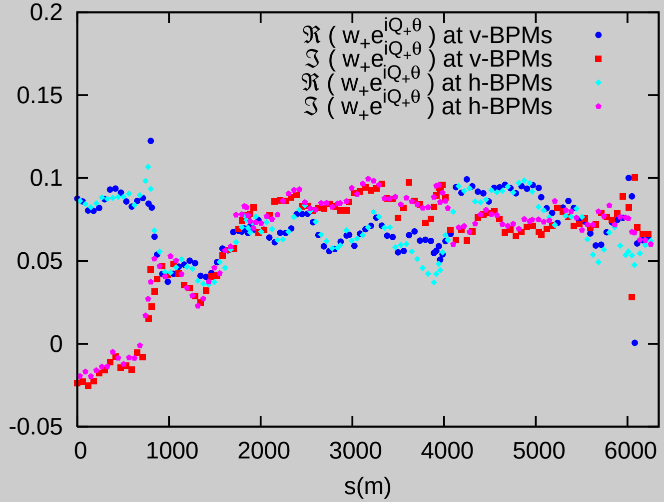


Horizontal

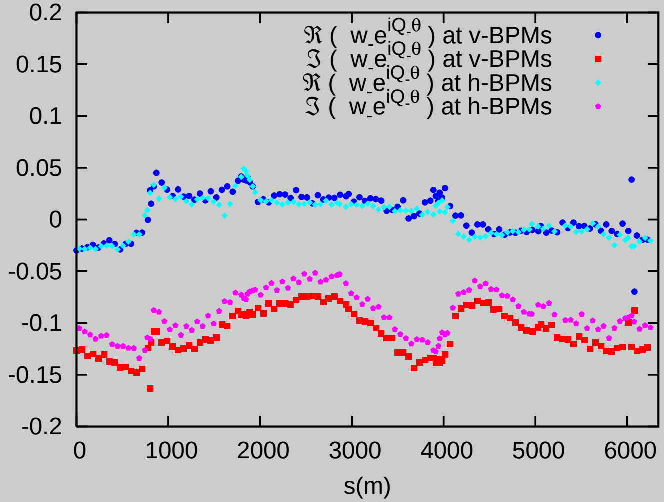


Vertical

Coupling functions (2006)

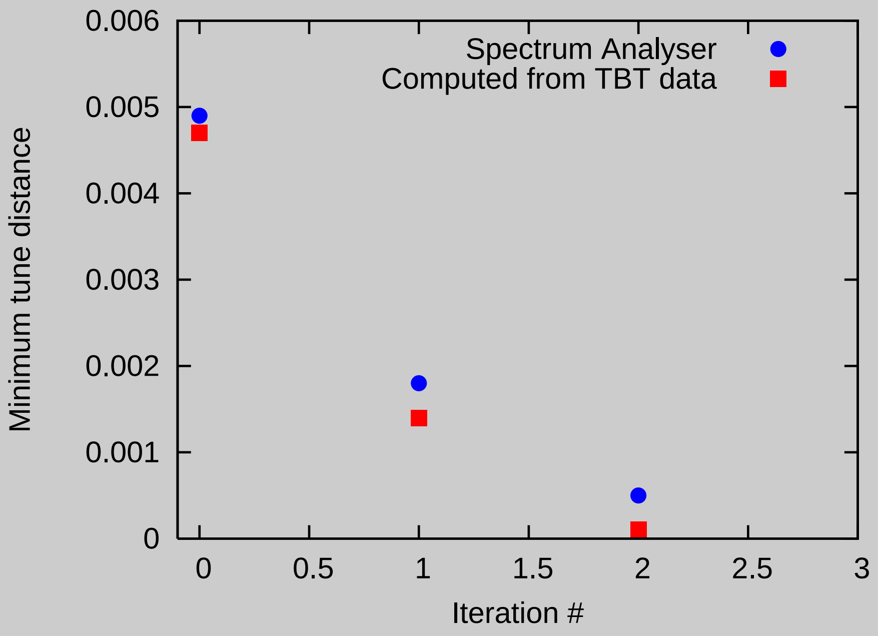


$\tilde{w}^{+}$



$\tilde{w}^{-}$

Minimum tune split measured with S.A. and computed from TBT data



- Spectrum Analyser
- TBT Analysis

EFFECT OF ERRORS

BPM's calibration errors affect the value of β_z^j computed through the Fourier analysis.

The effect of random calibration errors results in a unphysical beta-beating which is likely to average away when computing the oscillation amplitude.

A systematic calibration error has no effect on the evaluation of the β functions, but results in a wrong estimate of the oscillation amplitude and therefore of $w_\pm^j$ (unless the error is the same, for both horizontal and vertical BPM's).

By requiring

$$M_{12}^{meas} = M_{12}^{theo} \quad (\text{or } M_{34}^{meas} = M_{34}^{theo})$$

one can compute β_z^j resorting only on the measured phase advance. This requires (at least) three (consecutive) BPM's. Comparison with the value computed through the Fourier analysis may be used to calibrate the BPM's involved. However this will *not* correct for a possible systematic calibration error.

Through simulations we have estimated that the error on the evaluation of $|\bar{C}_-|$ is

- $\sim$ 2.5% for 5% systematic calibration error of either horizontal or vertical BPM's (they cancel out when the error has the *same* value)
- 0.5% for 5% random calibration errors
- a systematic tilt by 1^0 of all BPM's results in a error $|\delta\bar{C}_-| \simeq 0.0002$
- the error due to random tilts is negligible.

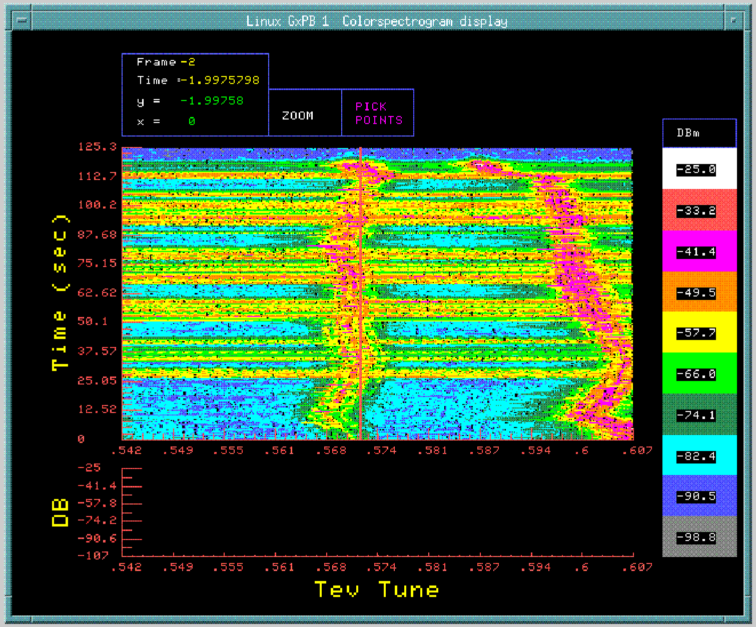
An application program for the TBT analysis has been integrated in the TEVATRON control system and is used routinely at shot set up for correcting the linear coupling. The correction is accurate and very fast compared to the usual method of finding the minimum tune split by empirical adjustement of the skew quadrupole circuits.

The time needed to retrieve the data has been greatly improved[a]. Although now we could use more BPM's also for routine operation we kept the old scheme, just 5 horizontal and 5 vertical BPM's, the TEVATRON working point (Q_x=20.584 and Q_z=20.574) being close to $Q_x \pm Q_y = int$. Off-line analysis using all BPM's has shown only little differences.

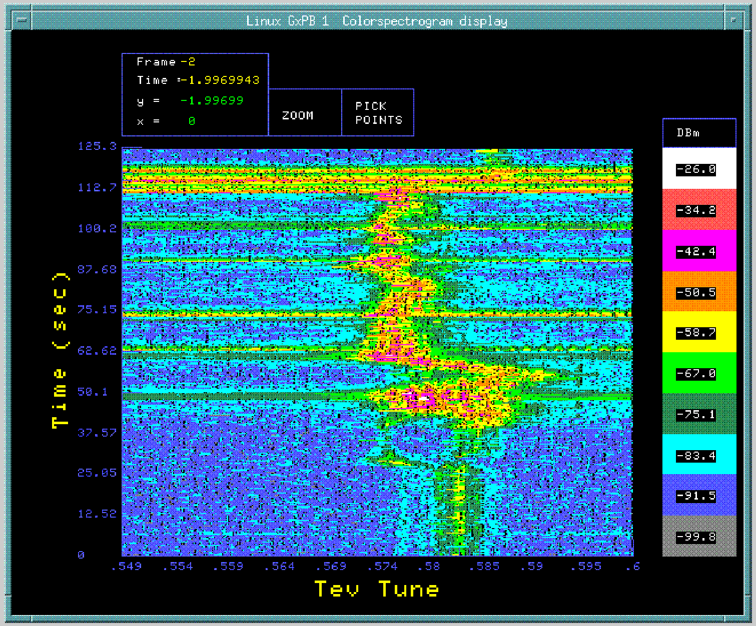
TEVATRON being a fast ramping machine (83 seconds from 150 to 980 GeV), the TBT analysis is the only practical method for measuring optics and coupling also during acceleration. The application has been used several times, in particular after long shut-downs, for decoupling on the ramp.

[a] it used to be for instance 7 minutes for 256 turns and 236 BPM's

Tunes on ramp after 2006 shut-down



Very first ramp before correction



...after correction

FIT OF TBT DATA USING MAD-X

The Fourier analysis of the measured TBT data

$$\begin{aligned} x_n = & A_I\sqrt{\beta_{xI}}\cos(\phi_{xI} + \delta_I + 2\pi n Q_I) + \\ & A_{II}\sqrt{\beta_{xII}}\cos(\phi_{xII} + \delta_{II} + 2\pi n Q_{II}) \\ y_n = & A_I\sqrt{\beta_{yI}}\cos(\phi_{yI} + \delta_I + 2\pi n Q_I) + \\ & A_{II}\sqrt{\beta_{yII}}\cos(\phi_{yII} + \delta_{II} + 2\pi n Q_{II}) \end{aligned}$$

gives the coupled Mais-Ripken twiss functions $\beta_{zI,II}$ and $\phi_{zI,II}$ $(z \equiv x, y)$, a part for the constants of motion $A_{I,II}$ and $\delta_{I,II}$.

The eigenvectors of the coupled transport matrix are related to the Mais-Ripken twiss functions

$$
\begin{aligned}
\boldsymbol{V_{11}} &\equiv \sqrt{\boldsymbol{\beta}_{xI}}\,\cos\boldsymbol{\phi}_{xI} & \boldsymbol{V_{12}} &\equiv \sqrt{\boldsymbol{\beta}_{xI}}\,\sin\boldsymbol{\phi}_{xI}\\
\boldsymbol{V_{13}} &\equiv \sqrt{\boldsymbol{\beta}_{xII}}\,\cos\boldsymbol{\phi}_{xII} & \boldsymbol{V_{14}} &\equiv \sqrt{\boldsymbol{\beta}_{xII}}\,\sin\boldsymbol{\phi}_{xII}\\
\boldsymbol{V_{31}} &\equiv \sqrt{\boldsymbol{\beta}_{yI}}\,\cos\boldsymbol{\phi}_{yI} & \boldsymbol{V_{32}} &\equiv \sqrt{\boldsymbol{\beta}_{yI}}\,\sin\boldsymbol{\phi}_{yI}\\
\boldsymbol{V_{33}} &\equiv \sqrt{\boldsymbol{\beta}_{yII}}\,\cos\boldsymbol{\phi}_{yII} & \boldsymbol{V_{34}} &\equiv \sqrt{\boldsymbol{\beta}_{yII}}\,\sin\boldsymbol{\phi}_{yII}
\end{aligned}
$$

Taking into account that the BPMs may have (unknown) calibration errors and may be tilted around the longitudinal axis[a] the actual eigenvector components are related to the measured ones, $\bar{V}^i_{lk}$ ($i \equiv$ BPM index), by

$$
\begin{gathered}
\frac{1}{A_I}[\cos(\delta_I)\bar{V}^i_{11} + \bar{V}^i_{12}\sin(\delta_I)] = \frac{1}{r_i}V^i_{11} + \frac{\chi_i}{r_i}V^i_{31} \\
\frac{1}{A_I}[-\sin(\delta_I)\bar{V}^i_{11} + \bar{V}^i_{12}\cos(\delta_I)] = \frac{1}{r_i}V^i_{12} + \frac{\chi_i}{r_i}V^i_{32} \\
\frac{1}{A_{II}}[\cos(\delta_{II})\bar{V}^i_{13} + \bar{V}^i_{14}\sin(\delta_{II})] = \frac{1}{r_i}V^i_{13} + \frac{\chi_i}{r_i}V^i_{33} \\
\frac{1}{A_{II}}[-\sin(\delta_{II})\bar{V}^i_{13} + \bar{V}^i_{14}\cos(\delta_{II})] = \frac{1}{r_i}V^i_{14} + \frac{\chi_i}{r_i}V^i_{34} \\
\frac{1}{A_I}[\cos(\delta_I)\bar{V}^i_{31} + \bar{V}^i_{32}\sin(\delta_I)] = \frac{1}{r_i}V^i_{31} - \frac{\chi_i}{r_i}V^i_{11} \\
\frac{1}{A_I}[-\sin(\delta_I)\bar{V}^i_{31} + \bar{V}^i_{32}\cos(\delta_I)] = \frac{1}{r_i}V^i_{32} - \frac{\chi_i}{r_i}V^i_{12} \\
\frac{1}{A_{II}}[\cos(\delta_{II})\bar{V}^i_{33} + \bar{V}^i_{34}\sin(\delta_{II})] = \frac{1}{r_i}V^i_{33} - \frac{\chi_i}{r_i}V^i_{13} \\
\frac{1}{A_{II}}[-\sin(\delta_{II})\bar{V}^i_{33} + \bar{V}^i_{34}\cos(\delta_{II})] = \frac{1}{r_i}V^i_{34} - \frac{\chi_i}{r_i}V^i_{14}
\end{gathered}
$$

[a] The BPM reading is related to the actual beam position by

$$
x^{meas} = \frac{x + y\tan\chi}{r_x} \qquad y^{meas} = \frac{y - x\tan\chi}{r_y}
$$

with $\chi \equiv$ BPM tilt and $r_z \equiv z/z^{meas}$ ($z \equiv x, y$).

Goal: adjust

- quadrupole gradient and tilt
- BPMs calibration and tilt
- $A_{I,II}$ and $\delta_{I,II}$

in order to fit the measured eigenvector values at the BPMs.

It could be a good alternative to time consuming (for both data taking and computation) Orbit Response Matrix methods currently used.

MAD-X is capable of matching *coupled* optics and allows *user-defined* expressions in matching constraints ("macros").

MAD-X TWISS uses Edwards-Teng formalism.
MAD-X PTC_TWISS uses Mais-Ripken formalism, but it is too slow for matching purposes.

The two formalism are of course related, the relationships between the two sets of twiss functions being

$$\beta_{xI} = \kappa\beta_1 \quad \beta_{yII} = \kappa\beta_2 \qquad \phi_{xI} = \varphi_1 \quad \phi_{yII} = \varphi_2$$
$$\beta_{xII} = \kappa[R_{22}(R_{22}\beta_2 + 2R_{12}\alpha_2) + R_{12}^2\gamma_2]$$
$$\beta_{yI} = \kappa[R_{11}(R_{11}\beta_1 - 2R_{12}\alpha_1) + R_{12}^2\gamma_1]$$
$$\phi_{xII} = \varphi_2 - \arctan[R_{12}/(R_{22}\beta_2 + R_{12}\alpha_2)]$$
$$\phi_{yI} = \varphi_1 + \arctan[R_{12}/(R_{11}\beta_1 - R_{12}\alpha_1)]$$

with $\kappa \equiv 1/(1+|R|)$, R being a 2×2 matrix, also computed by MAD-X.

Use MAD-X macros to define

- Mais-Ripken functions in terms of Edwards-Teng ones
- constraints & variables

Application to Tevatron

- Number of observation points: 2×118
- Current Tevatron model (A.Valishev): 216 normal and 216 skew thin quadrupoles to simulate gradient and tilt errors. We must add the unknown BPM calibrations and tilts, with the additional condition $< r_i >=1$, and the oscillation amplitude and phase.

All together: 908 parameters and 945 constraints. The fit is time consuming, therefore

- simplify input by lumping elements and declare them as one MATRIX
- split fit
  - use MADX for fitting magnetic elements
  - fit linearly BPMs calibration and tilt by a different code

  and iterate

But

- Still too slow for a console application $\longrightarrow$ write our own minimisation code
- Problems by fitting real data (July 07): no convergence (preliminary).

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